Mean-Variance-Skewness-Kurtosis Efficiency of Portfolios Computed via Moment-based Bounds

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Abstract—We analyze moment-based bounding approximations on the expected value of a utility function. We show that optimizing these bounds yields a solution, which is mean-variance (MV) or MV-skewness-kurtosis (MVSK) efficient depending on how many moments are included in the approximation. To illustrate the approach we apply it to an asset allocation model with a shortfall utility function. Numerical results are presented for an out of sample trading strategy using sixteen years of daily trading for a portfolio of six assets. The strategy significantly outperforms a standard market index, Dow Jones Industrial Average.

I. INTRODUCTION

Moment-based approximations on the expected value of function of a random variable have a long history in approximately solving stochastic optimization models; see, e.g., [1]–[6] and references therein. In this paper we apply moment-based approximations, using up to four moments, to an asset allocation model. We establish conditions under which notions of efficient portfolios are obtained. When optimizing a moment-based approximation of expected utility involving only the first two moments, we analyze whether the property of mean-variance (MV) efficiency is satisfied. This idea is extended to the notion of MV-skewness-kurtosis (MVSK) efficient portfolios when using the first four moments. MV efficiency is well-known since Markowitz’s seminal work on portfolio selection [7]. MVSK efficiency has received attention in more recent papers; e.g., see [8], [9].

The approximations we study are deterministically valid lower and upper bounds on the unknown expected value of a model’s objective function. Bounding approximations are used because the expected value is a multidimensional integral that is difficult to compute.

The rest of this paper is organized as follows. Section II reviews the MV efficiency of a second-order bound on the utility function. This bound is from a decreasing sequence of higher-order upper bounds on the expectation of a convex function [10]. Section II also describes the extension of MV efficiency to MVSK efficiency using a fourth-order bound on a utility function. Section III presents our empirical results with respect to a six-asset portfolio. The MV and MVSK efficient portfolio weights—recalculated daily—are utilized in a backtest of trading over fifteen years. Finally, Section IV offers conclusions and directions for future research.

II. EFFICIENCY OF TWO MOMENT-BASED BOUNDS

Portfolio optimization problems are extensively studied by practitioners and in the scientific literature. It is widely accepted that portfolio theory began in the 1950s with the seminal quadratic optimization model of [7] for minimizing the variance subject to a specified level of mean portfolio return. More recent work has included the notion of coherent measures of risk [11] (which includes conditional value-at-risk [12]) and time-dynamic versions of such risk measures [13], [14].

We consider a single-period asset allocation model formulated as

\[
\max \ E_u(x^\top \xi) \quad \text{s.t.} \quad x \in X, \tag{1}
\]

where \( u \) is an increasing, concave utility function. The assets have random simple gross returns, denoted \( \xi_i, i = 1, \ldots, m \), for stocks that are assumed to pay no dividends. Decision variable \( x \) denotes the proportion of our portfolio we decide to invest in asset \( i \), and we let \( x^\top = (x_1, \ldots, x_m) \). The set \( X \) consists of portfolio weights such that

\[
x \in X = \{x : x^\top 1 = 1, \ x \geq 0\},
\]

where \( 1 \) is a vector of ones. Let the vector of random returns, \( \xi = (\xi_1, \ldots, \xi_m)^\top \), have covariance matrix \( \Sigma \) and mean \( \mu \). A portfolio, \( p \), has random return \( r_p = x^\top \xi \) with mean return \( \mu = E(r_p) = x^\top \mu \), variance \( \sigma^2 = \text{Var}(r_p) = E((r_p - \mu)^2) \), skewness represented by the third central moment \( \text{Skew}(r_p) = E((r_p - \mu)^3)/\sigma^3 \), and kurtosis \( \text{Kurt}(r_p) = E((r_p - \mu)^4)/\sigma^4 \) by the fourth central moment, where the last two moments are normalized by the variance.

We note that although we deal with a random vector \( \xi \) of asset returns, the utility function \( u(x^\top \xi) \) depends only on the one-dimensional random variable, \( r_p \). Hence, we will utilize univariate moment-based bounds for a concave function of a random variable.

Solving the optimization model (1) is a difficult task because we have to know, or assume, a probability distribution under
which the expectation of the utility function $\mathbb{E} u(x^T \xi)$ is computed. However, if we replace this expectation by a univariate stochastic programming bound then we only have to know, or estimate, the first few moments of the random variable $r_p$. We do so in Sections II-A and II-B. In particular, we replace the objective function in (1) by the $EM_n$ bounds of [10]. The formulas developed therein (see Theorem 2, [10]) say that for a convex function $f : [a, b] \rightarrow \mathbb{R}$ and non-degenerate random variable $\eta$, with support contained in a finite interval $[a, b]$, and moments $\mathbb{E} \eta^i$, $i = 1, \ldots, n$, the expression

$$
\sum_{i=0}^{n} \binom{n}{i} \frac{\mathbb{E} [(\eta - a)^i (b - \eta)^{n-i}]}{(b-a)^n} f \left( \frac{a + i}{n} (b-a) \right) 
$$

$$
\equiv EM_n \geq EM_{n+1} \geq \mathbb{E} f(\eta),
$$

provides a decreasing sequence of upper bounds on the expectation $\mathbb{E} f(\eta)$. Technically, $EM_n$ is the expectation of the degree-$n$ Bernstein polynomial which approximates the given function $f$.

In order to apply the $EM_n$ bounds, and to replace the expectation $\mathbb{E} u(r_p)$ in (1) with maximization of $EM_n$ we require: (i) the function $u$ to be convex or concave, (ii) the moments of $r_p$ to be known, and (iii) the support of $r_p$ to be bounded.

In our empirical results of Section III, we consider a benchmark-based utility function of the following form:

$$
u(x^T \xi) = x^T \xi - \lambda [\mu_b - x^T \xi]^+.
$$

Under this utility, model (1) optimizes a weighted sum of expected return and a risk term for falling short of a benchmark, $\mu_b$. Larger values of $\lambda > 0$ correspond to increased weight on the risk term, i.e., increased risk aversion. Condition (i) above is satisfied by the utility of equation (3). We introduce this piecewise linear concave utility function now only for concreteness. Our MV and MVSK efficiency characterizations in this section hold for more general utilities.

Given $x$, $r_p = x^T \xi$ and so $Er_{p}^m = \mathbb{E} (\sum_{i=1}^{m} x_i \xi_i)^m$ can be computed by expanding the degree-$n$ polynomial and utilizing (assumed) knowledge of the associated cross-moments of $\xi_i$, $i = 1, \ldots, m$. Hence condition (ii) is satisfied because we have an explicit functional form for $Er_{p}^m = Er_{p}^m(x)$ as a degree-$n$ polynomial in $x$. In what follows we will assume that the vector of asset returns, $\xi$, has bounded support so that condition (iii) holds. In particular we will assume that $a_i \leq \xi_i \leq b_i$, w.p.1., (i.e., with probability one) for every asset $i = 1, \ldots, m$ and thus

$$
0 \leq a = \min_{1 \leq i \leq m} a_i \leq r_p(x) \leq \max_{1 \leq i \leq m} b_i = b.
$$

Note (4) holds regardless of the values of $x$ because $r_p$ is a convex combination of the $\xi_i$’s, and nonnegativity holds because we are using simple gross returns. When short-selling of stocks is permitted within bounds then each end of the finite support $[a, b]$ will depend on both max-and-min asset returns.

### A Second-Order Proxy Model

In this section we apply the univariate bound $EM_2$ stated in equation (2) to the general one-period asset allocation problem defined in model (1). The resulting objective function is denoted $EM_2(u(\cdot))$, and we view the associated approximation as a proxy model:

$$
\max_{x} \quad EM_2 (u(x^T \xi)) 
\text{ s.t. } x \in X.
$$

The set of all mean-variance efficient portfolios is called the efficient frontier $(EF)$. The formal definition of $EF$ follows.

**Definition 1.** The efficient frontier $(EF)$ of model (1) is:

$$
EF = \{ (\mu, \sigma^2) : \exists x \in X \text{ with } \mu = x^T \mu, \ \sigma^2 = x^T \Sigma x \text{ and } \beta \hat{x} \in X \text{ satisfying } \hat{x}^T \mu > \mu, \ \hat{x}^T \Sigma \hat{x} \leq \sigma^2 \}.
$$

An allocation $x \in X$ is said to be on the mean-variance efficient frontier provided $(x^T \mu, x^T \Sigma x) \in EF$.

The definition simply says that an allocation is mean-variance efficient if there is no other feasible allocation yielding larger expected return and smaller variance of return. We state Definition 1 in the context of model (1), but note that it does not depend on a specific asset allocation model.

We next show that the proxy model (5) yields mean-variance efficient solutions for any vector $\xi$ of random returns.

**Theorem 2.** Consider model (1), where $u : [a, b] \rightarrow \mathbb{R}$ is increasing, concave, and not linear, and assume condition (4) holds for all $x \in X$. Then an optimal solution of (5) yields a mean-variance efficient portfolio.

**Proof** Equation (2) with $n = 2$ yields

$$
EM_2 = \frac{(b - \mu)^2 + \sigma^2}{(b-a)^2} u(a) + 2 \frac{(b - \mu)(\mu - a) - \sigma^2}{(b-a)^2} u \left( \frac{a+b}{2} \right) + \frac{(\mu - a)^2 + \sigma^2}{(b-a)^2} u(b),
$$

where $\mu = Er_{p} = x^T \mu$, $\sigma^2 = \text{Var}(r_p) = x^T \Sigma x$, and $x \in X$. Thus $EM_2$ depends on $r_p$ only through $\mu$ and $\sigma^2$, and so it is sufﬁces to show

$$
\frac{\partial}{\partial \mu} (EM_2) > 0 \quad \text{and} \quad \frac{\partial}{\partial \sigma^2} (EM_2) < 0.
$$

To this end, we have

$$
\frac{\partial}{\partial \sigma^2} (EM_2) = \frac{1}{(b-a)^2} \left( u(a) - 2u \left( \frac{a+b}{2} \right) + u(b) \right) < 0
$$

$$
\frac{\partial}{\partial \mu} (EM_2) > 0 \quad \text{and} \quad \frac{\partial}{\partial \sigma^2} (EM_2) < 0.
$$

The set of all mean-variance efficient portfolios is called the efficient frontier $(EF)$. The formal definition of $EF$ follows.
because \( u \) is concave, and not linear, on \([a, b]\). And, we have

\[
\frac{\partial}{\partial \mu} (EM_2) = \frac{2}{(b-a)^2} \left[ -(b-\mu)u(a) + (a+b-2\mu)u \left( \frac{a+b}{2} \right) + (\mu-a)u(b) \right] \\
+ \frac{2}{(b-a)^2} \left[ (\mu-a) \left( u(b) - u \left( \frac{a+b}{2} \right) \right) \\
+ (b-\mu) \left( u \left( \frac{a+b}{2} \right) - u(a) \right) \right] > 0
\]

because \( u \) is increasing. \( \square \)

It is well known (e.g., [15]) that the asset allocation model (1) is mean-variance efficient under a quadratic utility function of the form \( u(r_p) = r_p - \lambda r_p^2 \) for \( \lambda > 0 \). Our \( EM_2 \) approximation is a quadratic function, and we may essentially view Theorem 2 as establishing that \( EM_2 (u (r_p)) \) can be expressed as \( \mathbb{E} r_p - \lambda \mathbb{E} r_p^2 \) with \( \lambda > 0 \).

As we see from equation (2), a tighter approximating model in (5) is provided by replacing \( EM_2(u) \) with \( EM_n(u) \) because \( EM_2(u) \leq EM_n(u) \leq \mathbb{E} u \) under a concave utility, where \( n \geq 2 \). We do so in next section with \( n = 4 \).

### B. A Fourth-Order Proxy Model

Here we study portfolio efficiency with respect to the first four moments of a portfolio’s return. And, we show that optimizing the fourth-order bounding approximation \( EM_4 \) leads to a portfolio that is MVSK efficient.

The importance of skewness in regression models similar to CAPM (capital asset pricing model) is widely recognized in the finance literature; see, e.g., [16]. A model maximizing skewness subject to first- and second-moment constraints is studied in [17]. The higher fidelity optimization model, which adds skewness-and-kurtosis efficiency is extensively studied in recent papers including [8] and [9]. Basically, an investor’s objective is to achieve simultaneously high return and high skewness coupled with low variance and low kurtosis. The following definition captures this idea.

**Definition 3.** The MVSK efficient frontier of model (1) is:

\[
\text{muskEF} = \{(\mu, \sigma^2, s, k) : \exists \bar{x} \in X \text{ with } r_p = \bar{x}^\top \xi, \\
\mu = \mathbb{E} r_p, \quad \sigma^2 = \text{Var}(r_p), \quad s = \text{Skew}(r_p), \quad k = \text{Kurt}(r_p) \}
\]

and \( \bar{x} \in X \) with \( r_p = \bar{x}^\top \xi \) satisfying

\[
\mathbb{E} r_p > \mu, \quad \text{Var}(\bar{r}_p) \leq \sigma^2, \quad \text{Skew}(\bar{r}_p) \geq s, \quad \text{Kurt}(\bar{r}_p) \leq k.
\]

An allocation \( \bar{x} \in X \) is said to be on the MVSK efficient frontier provided \( r_p = \bar{x}^\top \xi \) satisfies \( (\mathbb{E} r_p, \text{Var}(r_p), \text{Skew}(r_p), \text{Kurt}(r_p)) \in \text{muskEF} \).

Definition 3 mathematically characterizes the set of all portfolio allocations optimally selected with respect to the first four moments. A portfolio allocation is MVSK efficient if there is no other feasible allocation which has greater expected return combined with smaller variance, greater skewness and smaller kurtosis.

The following proposition provides sufficient conditions for ensuring a utility function in model (1) will yield a MVSK efficient solution.

**Proposition 4.** Assume \( Eu(r_p) \) in model (1) can be expressed as a function \( G(m_1, m_2, m_3, m_4) \), where \( m_i = \mathbb{E} r_p^i, \quad i = 1, 2, 3, 4 \). Then an optimal solution of model (1) is on the MVSK efficient frontier provided

\[
\begin{align*}
G & \text{ increases in } m_1, \\
G & \text{ decreases in } m_2, \\
G & \text{ increases in } m_3, \text{ and} \\
G & \text{ decreases in } m_4.
\end{align*}
\]

**Proof.** The proof is based on three facts. First, the monotonicity of \( G \) in \( m_4 \) is equivalent to the monotonicity of \( G \) in \( \text{Kurt}(r_p) \). Because \( \text{Kurt}(r_p) = (m_4 - 4m_3m_1 + 6m_2m_1^2 - 3m_1^3)/\sigma^4 \) increases if and only if \( m_4 \) increases when \( m_1, m_2, m_3 \) are fixed. Second, the monotonicity of \( G \) in \( m_3 \) is equivalent to the monotonicity of \( G \) in \( \text{Skew}(r_p) \). Because \( \text{Skew}(r_p) = (m_3 - 3m_2m_1 + 2m_1^2)/\sigma^3 \) increases if and only if \( m_3 \) increases when \( m_1, m_2 \) are fixed. And, third, \( G \) increases in \( m_2 \) if \( G \) increases in \( \text{Var}(r_p) = \sigma^2 = m_2 - m_1^2 \) provided \( m_1 \) is fixed. Let \( \bar{x}^* \) solve

\[
\max_{\bar{x} \in X} \{ Eu(\bar{x}^\top \xi) = G(m_1, m_2, m_3, m_4) \}
\]

with \( m_1^* = \mathbb{E} (r_p)^i, \quad i = 1, 2, 3, 4 \), and suppose \( \bar{x}^* \) is not MVSK efficient. Definition 3 implies there is \( \bar{x} \in X \) such that \( \mathbb{E} r_p > \mathbb{E} r_p^* \), \( \text{Var}(\bar{r}_p) \leq \text{Var}(r_p^*) \), \( \text{Skew}(\bar{r}_p) \geq \text{Skew}(r_p^*) \), and \( \text{Kurt}(\bar{r}_p) \leq \text{Kurt}(r_p^*) \), where \( \mathbb{E} (\bar{r}_p)^4 = \hat{m}_4 \). We complete the proof by observing the following contradiction between the optimality of \( \bar{x}^* \) and the existence of \( \bar{x} \), which provides a larger objective function value

\[
\max_{\bar{x} \in X} \{ EM_4(u(r_p)) \}
\]

s.t. \( \bar{x} \in X \),

where, by equation (2), \( EM_4 \)

\[
EM_4 = \sum_{i=0}^{4} \left( \frac{4}{i} \right) \mathbb{E} [(r_p - a)^i(b - r_p)^{4-i}] u \left( a + i(b - a)/4 \right).
\]

The following theorem provides sufficient conditions on \( u \) so that the fourth-order proxy model yields MVSK efficient solution.

**Theorem 5.** Consider model (1), where \( u : [a, b] \to \mathbb{R} \) is increasing, concave, and satisfies

\[
\begin{align*}
u_4 - 4u_3 + 6u_2 - 4u_1 + u_0 < 0 \quad (8a) \\
u_3 - 3u_2 + 3u_1 - u_0 > 0 \quad (8b)
\end{align*}
\]
where \( u_i = u(a + i(b - a)/4), i = 0, \ldots, 4 \). Assume condition (4) holds for all \( x \in X \). Then an optimal solution of (7) is on the MVSK efficient frontier.

**Proof** It suffices to show that the conditions in (6) hold for

\[
G(m_1, m_2, m_3, m_4) = EM_4(u(r_p)).
\]

In doing so we will prove that

\[
G_{m_4}' = \frac{\partial}{\partial m_4} (EM_4) < 0, \quad G_{m_3}' = \frac{\partial}{\partial m_3} (EM_4) \geq 0
\]

\[
G_{m_2}' = \frac{\partial}{\partial m_2} (EM_4) \leq 0, \quad G_{m_1}' = \frac{\partial}{\partial m_1} (EM_4) \geq 0.
\]

First, \( G_{m_4}' < 0 \) is equivalent to hypothesis (8a). Second, hypothesis (8a) may be rewritten

\[-u_4 + 3u_3 - 3u_2 + u_1 > -u_3 + 3u_2 - 3u_1 + u_0,
\]

and this coupled with \( 0 \geq -a > -b \) and condition (8b) implies

\[-a (-u_4 + 3u_3 - 3u_2 + u_1) \leq -a (-u_3 + 3u_2 - 3u_1 + u_0) \leq -b (-u_4 + 3u_3 - 3u_2 + u_1 + u_0),
\]

which is equivalent to \( G_{m_3}' \geq 0 \). Third, for \( G_{m_2}' \), let

\[
g_0 = u_2 - 2u_1 + u_0, \\
g_1 = u_3 - 2u_2 + u_1, \text{ and }
\]

\[
g_2 = u_4 - 2u_3 + u_2.
\]

Because of (8b) we have

\[g_1 - g_0 > 0,
\]

and because of (8a) we have

\[g_2 - g_1 < g_1 - g_0.
\]

Hence,

\[a^2 (g_2 - g_1) \leq a^2 (g_1 - g_0) \leq b^2 (g_1 - g_0),
\]

which is equivalent to

\[a^2 g_2 + b^2 g_0 \leq (a^2 + b^2) g_1.
\]

Because \( u \) is concave we have \( g_1 \leq 0 \), which coupled with \( a^2 + b^2 \geq 2ab \) implies

\[(a^2 + b^2) g_1 \leq 2ab g_1.
\]

Combining the last two inequalities yields

\[a^2 g_2 + b^2 g_0 \leq 2ab g_1,
\]

which is equivalent to \( G_{m_2}' \leq 0 \). Finally, in a similar fashion for \( G_{m_1}' \), we have from \( G_{m_2}' \leq 0 \) that

\[a^2 (u_3 - u_2) - 2ab (u_2 - u_1) + b^2 (u_1 - u_0) \geq
\]

\[a^2 (u_4 - u_3) - 2ab (u_3 - u_2) + b^2 (u_2 - u_1),
\]

and hence,

\[a (a^2 (u_3 - u_2) - 2ab (u_2 - u_1) + b^2 (u_1 - u_0)) \geq
\]

\[a (a^2 (u_4 - u_3) - 2ab (u_3 - u_2) + b^2 (u_2 - u_1)).
\]

We also have

\[a^2 g_1 \geq a^2 g_0 \geq b^2 g_0,
\]

which is equivalent to

\[a^2 (u_3 - u_2) + b^2 (u_1 - u_0) \geq (a^2 + b^2)(u_2 - u_1).
\]

Next, \( u_2 - u_1 > 0 \) because \( u \) is increasing and we thus have

\[(a^2 + b^2)(u_2 - u_1) \geq 2ab(u_2 - u_1)
\]

Combining the last two inequalities yields

\[a^2 (u_3 - u_2) - 2ab (u_2 - u_1) + b^2 (u_1 - u_0) \geq
\]

\[a (a^2 (u_3 - u_2) - 2ab (u_2 - u_1) + b^2 (u_1 - u_0)).
\]

Combining (9) and (10) proves \( G_{m_1}' \geq 0 \).

In order to gain additional insight into hypotheses (8a) and (8b) of Theorem 5 we assume that the utility function \( u \) is differentiable on \([a, b]\), \( u' \) is strictly convex, and \( u'' \) is strictly concave. Then with \( h > 0 \), consider the following function

\[g(a) = u(a + 2h) - 2u(a + h) + u(a)
\]

which has negative second derivative

\[g''(a) = u''(a + 2h) - 2u''(a + h) + u''(a) < 0
\]

(because \( u'' \) is concave); i.e., \( g \) is concave. Therefore,

\[g(a) + g(a + 2h) < 2g(a + h)
\]

which is equivalent to (8a) by taking \( h = \frac{b - a}{4} \). A similar observation shows that the first derivative of \( g \) is positive

\[g'(a) = u'(a + 2h) - 2u'(a + h) + u'(a) > 0
\]

because \( u' \) is convex. Therefore \( g(a) \) is an increasing function, and this implies

\[g(a) < g(a + h),
\]

which is equivalent to (8b) with \( h = \frac{b - a}{4} \). We summarize these results in the following corollary.

**Corollary 6.** Consider model (1), where \( u : [a, b] \to \mathbb{R} \) is increasing, concave, and twice continuously differentiable. Assume condition (4) holds for all \( x \in X \). If \( u' \) is strictly convex, and \( u'' \) is strictly concave then (8a) and (8b) hold, and so an optimal solution of (7) is on the MVSK efficient frontier.

MVSK efficiency is implied by \( u' > 0 \), \( u'' < 0 \), \( u''' > 0 \), and \( u'''' < 0 \) because the hypothesis of Corollary 6 are satisfied by a utility function having derivatives with such signs. These four inequalities are implied by decreasing risk aversion and decreasing absolute prudence in [18]. For another discussion on kurtosis and preferences for moments, see [19].

Next we present numerical results for a backtest when trading according to an asset allocation derived from solving optimization models (5) and (7).
We test the performance of a long/short dollar neutral trading strategy, which uses models (5) and (7). We use a utility function that corresponds only to the risk shortfall term in equation (3). We collected daily open and closing prices for four Exchange Traded Funds and two stocks from https://www.google.com/finance for the period 8/23/2001 to 07/19/2017. The selected tickers (unique identifiers) are: EFA, XLK, XLP, XLU, XOM, KO.

Every day after market close we use the last 42 days of daily returns to estimate the first four moments, including cross moments, for all assets. The next morning we trade with market-on-open orders and at the end of the trading day liquidate all positions with market-on-close orders. No transaction costs are included in the numerical results. Our trading strategy’s daily allocation is determined in two steps.

1) Step 1: Solve models (5) and (7) to obtain allocations $x_2^*$ and $x_4^*$. Use $x_2^*$ as the initial solution for the non-convex optimization model (7) when finding $x_4^*$. Both models are solved via Matlab's fmincon function.

2) Step 2: Let $\xi$ denote the return on the last trading day.

a) If the optimal weights in $x_2^*$ and $x_4^*$ for an asset in Step 1 differ in sign, we fix the lower and upper bounds on this asset to be zero.

b) Add a constraint to ensure that the optimal portfolio value on yesterday’s return cannot be lower than returns from the two models; i.e., add the constraint: $\xi^T x \geq \min \{ \xi^T x_2^*, \xi^T x_4^* \}$.

c) If $\xi^T x_2^* < \xi^T x_4^*$ then solve (5) else solve (7).

Step 2 in the above trading strategy is designed so that the strategy bets on mean reversion and a watermark-like bound is placed on the value of the optimal portfolio.

In this paper we study mean-variance (MV) and MV-skewness-kurtosis (MVK) efficient portfolios. Two moment-based bounds provide proxy models for the classical maximum utility model. We showed that the optimal solutions of the proxy models are either MV or MVSK efficient. These optimal solutions yield a strategy, which performs well in a backtest, trading at market-on-open and market-on-close.

Further research could analyze a strategy’s performance when positions are held overnight or longer. Another area of future research would analyze the MVSK efficiency of other available moment-based bounds.

References