NAME: 2020 IUT Admission Test Answer Sheet (SOCIE Math)

1. [10 points]

Squaring the equation
$\sqrt{8-x}=4-\sqrt{x+2}$, we get
$4\sqrt{x+2}=5+x$.
Squaring again, we get $x^2-6x-7=0$.
Therefore $x=-1,\ 7$, and the answer is 6.

Answer: 6

2. [10 points]

Multiply $(n-3)^2$ to both sides and then we get
$(n-2)(n-3)(n-5) \le 0$.
Thus $n \le 2,\ \ 3 < n \le 5$ since $n \ne 3$.
we have $n=1,2,4,5$
and $1+2+4+5=12$.

Answer: 12

3. [10 points]

The given equation is
$\log_2 \frac{x^2+5x+12}{x+5} = \log_2 2$, which leads to
$\frac{x^2+5x+12}{x+5} = 2$. Solving this equation, we get
$x^2+5x+12 = 2(x+5)$, which is
$x^2+3x+2 = (x+2)(x+1) = 0$. The solutions are $x = -2,\ -1$. Since these two solutions satisfy the given equation, they are genuine solutions. Hence, the sum of two solutions is -3 .

Answer: -3

4. [20 points]

From $\lim_{x\to\pi} \frac{f(x)}{x-\pi} = 3$, we have $f(\pi)=0$ and $f'(\pi)=3$. We note that

$$\lim_{x\to\pi} \frac{\sin(2f(x))}{x-\pi}$$

$$= \lim_{x\to\pi} \frac{\sin(2f(x))}{2f(x)} \times \frac{2(f(x)-f(\pi))}{x-\pi}$$

$$= 1\times 2f'(\pi) = 2\times 3 = 6.$$

Answer: 6

5. [20 points]

Note that $\int_0^{\pi} |x\cos x|\,dx =$

$$\int_0^{\frac{\pi}{2}} x\cos x\ dx - \int_{\frac{\pi}{2}}^{\pi} x\cos x\ dx$$

Using integration by parts, we get

$$\int x\cos x\,dx = x\sin x - \int \sin x\,dx$$

$$= x\sin x + \cos x + C.$$

Hence, $\int_0^{\pi} |x\cos x|\,dx$

$$= [x\sin x + \cos x]_0^{\frac{\pi}{2}} - [x\sin x + \cos x]_{\frac{\pi}{2}}^{\pi}$$

$$= \left(\frac{\pi}{2}-1\right) - \left(-1-\frac{\pi}{2}\right) = \pi$$

Answer: π

6. [30 points]

Putting $\int_0^{\pi} tf'(t) = C$, then $f(x)=\cos\frac{x}{2}+C$.

Hence, $f'(x) = -\frac{1}{2}\sin\frac{x}{2}$ and

$$C = \int_0^{\pi} tf'(t)\,dt = \int_0^{\pi} t\left(-\frac{1}{2}\sin\frac{t}{2}\right)dt$$

$$= \left[t\cos\frac{t}{2}\right]_0^{\pi} - \int_0^{\pi} \cos\frac{t}{2}\,dt$$

$$= -\left[2\sin\frac{t}{2}\right]_0^{\pi} = -2.$$

Hence,

$$f\left(\frac{\pi}{2}\right) = \cos\frac{\pi}{4} + C = \cos\frac{\pi}{4} - 2 = \frac{\sqrt{2}-4}{2}$$

Answer: $\frac{\sqrt{2}-4}{2}$