NAME: 2020 IUT Admission Test Answer Sheet (SOL Math)

1. [10 points]

One may use the given recursive formula to obtain

$a_2 = \frac{1}{a_1+1} = \frac{1}{2}$, $a_3 = \frac{1}{a_2+1} = \frac{2}{3}$,

$a_4 = \frac{1}{a_3+1} = \frac{3}{5}$ and $a_5 = \frac{1}{a_4+1} = \frac{5}{8}$

successively.

Answer: $\frac{5}{8}$

2. [10 points]

$\left(a+\frac{3}{a}\right)\left(3a+\frac{1}{a}\right) = 3a^2 + \frac{3}{a^2} + 10$

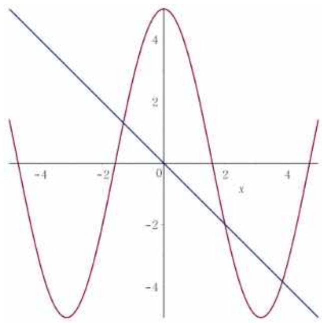
$= 3(a^2 + \frac{1}{a^2}) + 10 \geq 6\sqrt{a^2 \cdot \frac{1}{a^2}} + 10 = 16$.

Therefore the minimum value of $\left(a+\frac{3}{a}\right)\left(3a+\frac{1}{a}\right)$ is 16.

Answer: 16

3. [10 points]

Graphs of the two curves of of $y = 5\cos x$ and $y = -x$ are drawn as follows.



The two curves meets at 3 points.

Answer: 3

4. [20 points]

Since $\omega^2 = \omega + 1$, we have

$\omega^5 - 5\omega = (\omega+1)^2\omega - 5\omega$

$= \omega^3 + 2\omega^2 + \omega - 5\omega$

$= (\omega+1)\omega + 2(\omega+1) + \omega - 5\omega$

$= \omega^2 - \omega + 2 = (\omega+1) - \omega + 2 = 3$

Answer: 3

5. [20 points]

The line ℓ should be of the form $y = m(x-2)$. Moreover, $2x^2 - 3x = m(x-2)$ has a multiple root. Therefore, the determinant D of the equation $2x^2 - (3+m)x + 2m = 0$ should be zero. Thus, $D = m^2 - 10m + 9 = 0$. Therefore $m = 1 \text{ or } 9$. So, the least slope is 1.

Answer: 1

6. [30 points]

$\lim_{n\to\infty} \ln\left(\frac{3^{\frac{1}{n}} + 4^{\frac{1}{n}}}{2}\right)^n = \lim_{n\to\infty} \frac{\ln\left(3^{\frac{1}{n}} + 4^{\frac{1}{n}}\right) - \ln 2}{\frac{1}{n}}$

$= \lim_{x\to 0} \frac{\ln(3^x + 4^x) - \ln 2}{x} = \frac{d}{dx}\ln(3^x + 4^x)|_{x=0}$

$= \ln\sqrt{12}$

Thus $\lim_{n\to\infty}\left(\frac{3^{\frac{1}{n}} + 4^{\frac{1}{n}}}{2}\right)^n = \sqrt{12}$.

Therefore, the largest integer which is not exceeding $\sqrt{12}$ is 3.

Answer: 3