# 2021 IUT Admission Test(SOCIE, Contracted-Based)

# Math Examination

**<Essay Types>** Applicants should write detailed solving process. If there is no solution, you will receive 0 points regardless of the correct answer.

○ The point for each question is indicated next to each question number.

1. [5 points]

When $\alpha = \dfrac{5}{\sqrt{8}+\sqrt{3}}$ and $\beta = \dfrac{5}{\sqrt{8}-\sqrt{3}}$, find $\alpha^3+\beta^3$.

2. [5 points]

When $\sin\alpha + \cos\alpha = \dfrac{1}{4}$ for $0 \le \alpha \le \dfrac{\pi}{4}$, find $\sin\alpha - \cos\alpha$.

3. [5 points]

Evaluate $\displaystyle\sum_{n=1}^{100}\left(\frac{1+i}{1-i}\right)^n$.

4. [5 points]

Find the sum of all integers $a$ such that $3^{2x}-3^{x+1}+a=0$ has two distinct real solutions.

5. [10 points]

When $f(x) = \dfrac{\sin(x^2)}{x}$, find $f'(\sqrt{\pi})$.

6. [20 points]

Evaluate $\displaystyle\int_1^2 x^3\sqrt{x^2-1}\,dx$.

7. [20 points]

Find the area of the region enclosed by $y = x^2 + x$ and $y = -x^2 + 3x$.