The Solution of SOCIE Sample Test

1. Compute $\log_3\sqrt{108}+\log_{\frac{1}{9}}4+2^{\log_{\frac{1}{2}}3}$.

Sol) $\log_3\sqrt{108}+\log_{\frac{1}{9}}4+2^{\log_{\frac{1}{2}}3}$

$=\log_3(2^2 3^3)^{\frac{1}{2}}+\log_{3^{-2}}2^2+2^{\log_{2^{-1}}3}$

$=\log_3 2\cdot 3^{\frac{3}{2}}+\frac{2}{-2}\log_3 2+2^{\log_2 3^{-1}}$

$=\log_3 2\cdot 3^{\frac{3}{2}}-\log_3 2+3^{-1}$

$=\log_3 3^{\frac{3}{2}}+\frac{1}{3}=\frac{3}{2}+\frac{1}{3}=\frac{11}{6}$.

The answer is ③.

2. Compute $\sin\frac{13\pi}{12}$.

Sol) $\sin\left(\frac{13\pi}{12}\right)=\sin\left(\frac{\pi}{3}+\frac{3\pi}{4}\right)$

$=\sin\frac{\pi}{3}\cos\frac{3\pi}{4}+\cos\frac{\pi}{3}\sin\frac{3\pi}{4}$

$=\frac{1}{2}\cdot\frac{\sqrt{2}}{2}+\frac{\sqrt{3}}{2}\cdot\frac{-\sqrt{2}}{2}$

$=\frac{\sqrt{2}-\sqrt{6}}{4}$.

The answer is ②.

3. When $\begin{pmatrix}1 & 3\\ 0 & -1\end{pmatrix}^{-1}\begin{pmatrix}-2 & 2\\ 1 & 1\end{pmatrix}=\begin{pmatrix}a & b\\ c & d\end{pmatrix}$, find $a+b+c+d$.

Sol) $\begin{pmatrix}1 & 3\\ 0 & -1\end{pmatrix}^{-1}\begin{pmatrix}-2 & 2\\ 1 & 1\end{pmatrix}=\begin{pmatrix}1 & 3\\ 0 & -1\end{pmatrix}\begin{pmatrix}-2 & 2\\ 1 & 1\end{pmatrix}=\begin{pmatrix}1 & 5\\ -1 & -1\end{pmatrix}$. Hence, $a+b+c+d=4$.

The answer is ①.

4. When $\omega=\frac{1+\sqrt{3}i}{\sqrt{2}}$, find ω^{30}.

Sol) Note that $\omega^2=\frac{-2+2\sqrt{3}i}{2}=-1+\sqrt{3}i$ and

$\omega^3=\omega\cdot\omega^2=\frac{1+\sqrt{3}i}{\sqrt{2}}\cdot(-1+\sqrt{3}i)=\frac{-1-3}{\sqrt{2}}=-2\sqrt{2}$.

Hence, $\omega^{30}=(\omega^3)^{10}=(-2\sqrt{2})^{10}=2^{10}\cdot\sqrt{2^{10}}=2^{15}$.

The answer is ④.

5. Find the maximum value of $f(x) = \dfrac{x}{x^2+3}$.

Sol) From $f'(x) = \dfrac{x^2+3-2x^2}{(x^2+3)^2} = \dfrac{3-x^2}{(x^2+3)^2}$, $f'(x) = 0$ at $x = -\sqrt{3}$ and $x = \sqrt{3}$. Since $f(x) < 0$ for $x < 0$ and $f(x) > 0$ for $x > 0$ and $\lim_{x \to \pm\infty} f(x) = 0$, $f(x)$ has a maximum value at $x = \sqrt{3}$, which is $f(\sqrt{3}) = \dfrac{\sqrt{3}}{3+3} = \dfrac{\sqrt{3}}{6}$.

The answer is ③.

6. Find the area of the region enclosed by $y = x^5 + 2x^4 + x^2 - 1$ and $y = x^5 + 2x^4 + x + 1$.

Sol) Since $(x^5+2x^4+x+1) - (x^5+2x^4+x^2-1) = -(x^2-x-2) = -(x+1)(x-2)$, two graphs meet at $x = -1$ and $x = 2$. Since $x^5+2x^4+x+1 \geq x^5+2x^4+x^2-1$ on the interval $[-1, 2]$, the area is given by

$$\int_{-1}^{2} \left((x^5+2x^4+x+1) - (x^5+2x^4+x^2-1) \right) dx = \int_{-1}^{2} -(x^2-x-2)\,dx$$

$$= \left[-\frac{1}{3}x^3 + \frac{1}{2}x^2 + 2x \right]_{-1}^{2} = \left(-\frac{8}{3}+2+4 \right) - \left(\frac{1}{3} + \frac{1}{2} - 2 \right) = \frac{9}{2} .$$

The answer is ⑤.