Solution to the Sample Test for SBL

1. Simplify $\log_2 3 \times \log_5 8 \times \log_9 5$

(Sol) $\log_2 3 \times \log_5 8 \times \log_9 5 = \log_2 3 \times (3\log_5 2) \times \frac{1}{2}\log_3 5 = \frac{3}{2}$.

The answer is ②.

2. Let $A = \begin{pmatrix} 1 & 2 \\ -1 & 3 \end{pmatrix}$ and $B = \begin{pmatrix} -1 & 1 \\ 2 & 2 \end{pmatrix}$. When $A(B^{-1})^2 = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$, find $a+b+c+d$.

(Sol) Since $B^{-1} = \frac{1}{-4}\begin{pmatrix} 2 & -1 \\ -2 & -1 \end{pmatrix}$ and $B^{-2} = \frac{1}{16}\begin{pmatrix} 6 & -1 \\ -2 & 3 \end{pmatrix}$, we get

$AB^{-2} = \frac{1}{16}\begin{pmatrix} 1 & 2 \\ -1 & 3 \end{pmatrix}\begin{pmatrix} 6 & -1 \\ -2 & 3 \end{pmatrix} = \frac{1}{16}\begin{pmatrix} 2 & 5 \\ -12 & 10 \end{pmatrix}$. Hence, $a+b+c+d = \frac{5}{16}$.

The answer is ③.

3. When $\omega = \frac{1}{1+\sqrt{3}\,i}$, find ω^6.

(Sol) Since $(1+\sqrt{3}\,i)^3 = (1+\sqrt{3}\,i)^2(1+\sqrt{3}\,i) = -2(1-\sqrt{3}\,i)(1+\sqrt{3}\,i) = -8$, it follows that $w^6 = \frac{1}{(-8)^2} = \frac{1}{64}$.

The answer is ⑤.

4. Find $\cos\left(\frac{19\pi}{12}\right)$.

(Sol) Note that

$$\cos\left(\frac{19\pi}{12}\right) = \cos\left(\frac{5\pi}{12}\right) = \cos\left(\frac{\pi}{4}+\frac{\pi}{6}\right) = \cos\left(\frac{\pi}{4}\right)\cos\left(\frac{\pi}{6}\right) - \sin\left(\frac{\pi}{4}\right)\sin\left(\frac{\pi}{6}\right)$$
$$= \frac{\sqrt{2}}{2}\cdot\frac{\sqrt{3}}{2} - \frac{\sqrt{2}}{2}\cdot\frac{1}{2} = \frac{\sqrt{6}-\sqrt{2}}{4}.$$

The answer is ②.

5. When M and m are the maximum and minimum values of $f(x) = 2x^3 - 3x^2 - 12x + 5$, $(-2 \le x \le 2)$, find $M+m$.

(Sol)

Since $f'(x) = 6x^2 - 6x - 12 = 6(x^2 - x - 2) = 6(x-2)(x+1)$, $f'(x) = 0$ at $x = -1$ and $x = 2$. Since $f(-2) = -16 - 12 + 24 + 5 = 1$, $f(-1) = -2 - 3 + 12 + 5 = 12$ and $f(2) = 16 - 12 - 24 + 5 = -15$, the maximum value is 12 and the minimum value is -15. Hence, $M = 12$, $m = -15$, and $M + m = -3$.

The answer is ①.

6. Find $\int_0^1 x^2 (2x^3+1)^3\, dx$.

(Sol) Setting $u = 2x^3+1$, we get $du = 6x^2\, dx$.
It follows that

$$\int_0^1 x^2 (2x^3+1)^3\, dx = \frac{1}{6}\int_1^3 u^3\, du = \left[\frac{1}{24}u^4\right]_1^3 = \frac{27}{8} - \frac{1}{24} = \frac{10}{3} \ .$$

The answer is ④.