2021 IUT Admission Test (SBL, Type A)

Math Examination

<Essay Types> Applicants should write detailed solving process. If there is no solution, you will receive 0 points regardless of the correct answer.

○ The point for each question is indicated next to each question number.

1. [5 points]

   When $\sqrt{\dfrac{\sqrt[3]{3}}{\sqrt[4]{3}}}$ is written by $\sqrt[a]{b}$, find $a+b$.

2. [5 points]

   Evaluate $\int_0^1 x(x^2+1)^9 dx$.

3. [5 points]

   Find $\sum_{n=1}^{\infty} \dfrac{1}{n\sqrt{n+1}+(n+1)\sqrt{n}}$.

4. [5 points]

   When $f'(1)=3$, find $\lim_{x\to 1} \dfrac{f(x^2)-f(1)}{x-1}$.

5. [10 points]

   Find the polynomial $f(x)$ satisfying

   $\lim_{x\to\infty} \dfrac{f(x)}{x^2+x+1}=2$, $\lim_{x\to 1} \dfrac{f(x)}{x^2-4x+3}=4$.

6. [10 points]

When α, β are solutions of $x^{\log_2 x} = 4x^2$, find $\alpha\beta$.

7. [10 points]

When $A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$, $B = \begin{pmatrix} 1 & -1 \\ 2 & 1 \end{pmatrix}$,

find $A^{-1}(3A+2B)B^{-1}$.

8. [10 points]

Find the maximum and minimum values of $f(x) = \dfrac{2x^2+3x+2}{x^2+1}$.

9. [20 points]

Find the volume of the solid obtained by revolving the region enclosed by $y = \sqrt{x}$ and $y = \dfrac{1}{2}x$ about the x-axis.

10. [20 points]

When $g(x)$ is the inverse function of $f(x) = x^3 + x + 1$, evaluate

$\displaystyle\int_0^3 f(x)dx + \int_{f(0)}^{f(3)} g(x)dx$.

2021 IUT Admission Test Solution (SBL, Type A)

1. When $\sqrt{\dfrac{\sqrt[3]{3}}{\sqrt[4]{3}}}$ is written by $\sqrt[a]{b}$, find $a+b$.

sol) Since $\sqrt{\dfrac{\sqrt[3]{3}}{\sqrt[4]{3}}} = \dfrac{3^{1/6}}{3^{1/8}} = 3^{1/24} = \sqrt[24]{3}$, it follows that $a=24$, $b=3$ and $a+b=27$.

2. Evaluate $\int_0^1 x(x^2+1)^9 dx$.

sol) $\int_0^1 x(x^2+1)^9 dx = \left[\dfrac{1}{20}(1+x^2)^{10}\right]_0^1 = \dfrac{1023}{20}$.

3. Find $\sum_{n=1}^{\infty} \dfrac{1}{n\sqrt{n+1}+(n+1)\sqrt{n}}$.

sol) Observe that $\dfrac{1}{n\sqrt{n+1}+(n+1)\sqrt{n}} = \dfrac{\sqrt{n+1}-\sqrt{n}}{\sqrt{n(n+1)}} = \dfrac{1}{\sqrt{n}} - \dfrac{1}{\sqrt{n+1}}$.

Then $\sum_{n=1}^{\infty} \dfrac{1}{n\sqrt{n+1}+(n+1)\sqrt{n}} = 1$.

4. When $f'(1)=3$, find $\lim_{x\to 1} \dfrac{f(x^2)-f(1)}{x-1}$.

sol) $\lim_{x\to 1} \dfrac{f(x^2)-f(1)}{x-1} = \lim_{x\to 1} \dfrac{f(x^2)-f(1)}{x^2-1} \times (x+1) = 2f'(1) = 6$.

5. Find the polynomial $f(x)$ satisfying
$\lim_{x\to\infty} \dfrac{f(x)}{x^2+x+1} = 2$, $\lim_{x\to 1} \dfrac{f(x)}{x^2-4x+3} = 4$.
sol) By the given conditions, we know that $f(x) = 2(x-1)(x+a)$.
Since $\lim_{x\to 1} \dfrac{f(x)}{x^2-4x+3} = \lim_{x\to 1} \dfrac{2(x-1)(x+a)}{(x-1)(x-3)} = -(1+a)$, we get $f(x) = 2(x-1)(x-5)$.

6. When α, β are solutions of $x^{\log_2 x} = 4x^2$, find $\alpha\beta$.
sol) Taking logarithm on both sides of the equation, we get
$(\log_2 x)^2 - 2\log_2 x - 2 = 0$. Since it has two roots α, β, we deduce that
$(\log_2 x)^2 - 2\log_2 x - 2 = (\log_2 x - \log_2 \alpha)(\log_2 x - \log_2 \beta)$.

Therefore $\log_2 \alpha + \log_2 \beta = 2$, so that $\alpha\beta = 4$.

7. When $A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$, $B = \begin{pmatrix} 1 & -1 \\ 2 & 1 \end{pmatrix}$, find $A^{-1}(3A+2B)B^{-1}$.

sol) Since $A^{-1} = -\frac{1}{2}\begin{pmatrix} 4 & -2 \\ -3 & 1 \end{pmatrix}$, $B^{-1} = \frac{1}{3}\begin{pmatrix} 1 & 1 \\ -2 & 1 \end{pmatrix}$, we have

$A^{-1}(3A+2B)B^{-1} = 3B^{-1} + 2A^{-1} = \begin{pmatrix} -3 & 3 \\ 1 & 0 \end{pmatrix}$.

8. Find the maximum and minimum values of $f(x) = \frac{2x^2+3x+2}{x^2+1}$.

sol) Since $f'(x) = \frac{3(x^2+1) - 3x \cdot 2x}{x^2+1} = -\frac{3(x-1)(x+1)}{x^2+1}$, $f(x)$ is increasing for $-1 < x < 1$ and decreasing otherwise. We also observe $\lim_{x \to \pm\infty} f(x) = 2$. Therefore $f(x)$ has the maximum $\frac{7}{2}$ at $x = 1$ and the minimum $\frac{1}{2}$ at $x = -1$.

9. Find the volume of the solid obtained by rotating the region enclosed by $y = \sqrt{x}$ and $y = \frac{1}{2}x$ about the x-axis.

sol) Two curves $y = \sqrt{x}$ and $y = \frac{1}{2}x$ meet at $x = 0, 4$. Since $0 \le \frac{x}{2} \le \sqrt{x}$ for $0 \le x \le 4$, the volume is given by

$$\int_0^4 \pi\left\{(\sqrt{x})^2 - \left(\frac{x}{2}\right)^2\right\}dx = \pi\int_0^4 \left(x - \frac{x^2}{4}\right)dx = \pi\left[\frac{x^2}{2} - \frac{x^3}{12}\right]_0^4 = \frac{8\pi}{3}.$$

10. When $g(x)$ is the inverse function of $f(x) = x^3 + x + 1$, evaluate $\int_0^3 f(x)dx + \int_{f(0)}^{f(3)} g(x)dx$.

sol) Observe that $\int_{f(0)}^{f(3)} g(x)dx = \int_0^3 g(f(t))f'(t)dt = \int_0^3 tf'(t)dt = [tf(t)]_0^3 - \int_0^3 f(t)dt$.

Therefore, $\int_0^3 f(x)dx + \int_{f(0)}^{f(3)} g(x)dx = 3f(3) = 93$.

2021 IUT Admission Test(SOCIE, Contract-Based, Type A)

Math Examination

<Essay Types> Applicants should write detailed solving process. If there is no solution, you will receive 0 points regardless of the correct answer.

○ The point for each question is indicated next to each question number.

1. [5 points]
When α and β are solutions of $2x^2+5x+2=0$, find $\dfrac{\alpha}{\beta^2}+\dfrac{\beta}{\alpha^2}$.

2. [5 points]
Find the sum of all solutions of $\sqrt{3}\sin x + \cos x = \sqrt{2}$ for $0 \le x \le 2\pi$.

3. [5 points]
Evaluate $\displaystyle\sum_{n=1}^{99}\left(\frac{1+i}{\sqrt{2}}\right)^{2n}$.

4. [5 points]
When $\log_a b = \dfrac{1}{2}$ and $2a+3b=20$ for $a>0$ and $b>0$, find ab.

5. [10 points]
Find $\displaystyle\lim_{x\to 0}\frac{\sqrt{x^2+4}-2}{x^3+2x^2}$.

6. [20 points]
Evaluate $\displaystyle\int_1^9 e^{2\sqrt{x}}\,dx$.

7. [20 points]
Find the area of the region enclosed by $y=x^4+3x^3-2x+1$ and $y=3x^3-2x+2$.

2021 IUT Admission Test (SOCIE, Contract-Based, Type A)

(1) When α and β are solutions of $2x^2+5x+2=0$, find $\frac{\alpha}{\beta^2}+\frac{\beta}{\alpha^2}$.

(SOL) : We note that $\alpha+\beta=-\frac{5}{2}$ and $\alpha\beta=1$. Hence,

$$\frac{\alpha}{\beta^2}+\frac{\beta}{\alpha^2} = \frac{\alpha^3+\beta^3}{\alpha^2\beta^2} = \frac{(\alpha+\beta)(\alpha^2-\alpha\beta+\beta^2)}{(\alpha\beta)^2} = \frac{(\alpha+\beta)((\alpha+\beta)^2-3\alpha\beta)}{(\alpha\beta)^2}$$

$$= -\frac{5}{2}\times\left(\frac{25}{4}-3\right) = -\frac{5}{2}\times\frac{13}{4} = -\frac{65}{8}.$$

(2) Find the sum of all solutions $\sqrt{3}\sin x + \cos x = \sqrt{2}$ for $0 \le x \le 2\pi$.

(SOL) : From $\frac{\sqrt{3}}{2}\sin x + \frac{1}{2}\cos x = \frac{\sqrt{2}}{2}$, we have $\sin\left(x+\frac{\pi}{6}\right) = \frac{\sqrt{2}}{2}$.

Since $0 \le x \le 2\pi$, it follows that $x+\frac{\pi}{6} = \frac{\pi}{4}, \frac{3\pi}{4}$. Hence, $x=\frac{\pi}{12}$ or $\frac{7\pi}{12}$.

The sum of two solutions is $\frac{\pi}{12}+\frac{7\pi}{12}=\frac{2\pi}{3}$.

(3) Evaluate $\sum_{n=1}^{99}\left(\frac{1+i}{\sqrt{2}}\right)^{2n}$.

(SOL) : Setting $\omega=\frac{1+i}{\sqrt{2}}$, it follows that $\omega^2=\left(\frac{1+i}{\sqrt{2}}\right)^2=\frac{2i}{2}=i$. Hence,

$$\sum_{n=1}^{99}\left(\frac{1+i}{\sqrt{2}}\right)^{2n} = \sum_{n=1}^{99} i^n = \frac{i(1-i^{99})}{1-i} = \frac{i(1-(-i))}{1-i} = \frac{i(1+i)}{1-i} = \frac{-1+i}{1-i} = -1.$$

(4) When $\log_a b=\frac{1}{2}$ and $2a+3b=20$ for $a>0$ and $b>0$, find ab.

(SOL) : Since $b=a^{\frac{1}{2}}$ or $a=b^2$, the equation $2a+3b=20$ is reduced to $2b^2+3b-20=0$. Since $(2b-5)(b+4)=0$ and $b>0$, it follows that $b=\frac{5}{2}$ and $a=b^2=\frac{25}{4}$. Hence, $ab=\frac{125}{8}$.

(5) Find $\lim_{x \to 0} \frac{\sqrt{x^2+4}-2}{x^3+2x^2}$.

(SOL) :

Note that

$$\lim_{x \to 0} \frac{\sqrt{x^2+4}-2}{x^3+2x^2} = \lim_{x \to 0} \frac{\left(\sqrt{x^2+4}-2\right)\left(\sqrt{x^2+4}+2\right)}{x^2(x+2)\left(\sqrt{x^2+4}+2\right)}$$

$$= \lim_{x \to 0} \frac{x^2+4-4}{x^2(x+2)\left(\sqrt{x^2+4}+2\right)} = \lim_{x \to 0} \frac{1}{(x+2)\left(\sqrt{x^2+4}+2\right)} = \frac{1}{8} .$$

(6) Evaluate $\int_1^9 e^{2\sqrt{x}}\, dx$.

(SOL) : Setting $t = \sqrt{x}$, then $dt = \frac{1}{2\sqrt{x}} dx = \frac{1}{2t} dx$ and hence $dx = 2t\, dt$.

It follows that

$$\int_1^9 e^{2\sqrt{x}}\, dx = \int_1^3 2t\, e^{2t}\, dt = \left[\, t\, e^{2t}\, \right]_1^3 - \int_1^3 e^{2t} dt = 3e^6 - e^2 - \left[\frac{1}{2} e^{2t}\right]_1^3$$

$$= 3e^6 - e^2 - \frac{1}{2} e^6 + \frac{1}{2} e^2 = \frac{5}{2} e^6 - \frac{1}{2} e^2 .$$

(7) Find the area of the region enclosed by $y = x^4 + 3x^3 - 2x + 1$ and $y = 3x^3 - 2x + 2$.

(SOL) : From the equation $x^4 + 3x^3 - 2x + 1 = 3x^3 - 2x + 2$, we have $x^4 - 1 = (x^2-1)(x^2+1) = (x-1)(x+1)(x^2+1) = 0$ and $x = 1,\ -1$. This means that two curves meet at $x = -1$ and $x = 1$. Since $3x^3 - 2x + 2 \geq x^4 + 3x^3 - 2x + 1$ on $[-1, 1]$, the area of the region is given by

$$\int_{-1}^1 \{(3x^3 - 2x + 2) - (x^4 + 3x^3 - 2x + 1)\}\, dx = \int_{-1}^1 (1 - x^4)\, dx = 2 \int_0^1 (1 - x^4)\, dx$$

$$= 2\left[x - \frac{1}{5} x^5\right]_{x=0}^1 = \frac{8}{5} .$$

2021 IUT Admission Test(SOCIE)

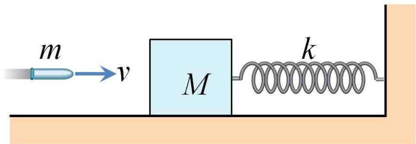
Physics Examination

<Essay Types> Applicants should write detailed solving process. If there is no solution, you will receive 0 points regardless of the correct answer.

○ The point for each question is indicated next to each question number.

○ Be sure to use SI units (the international system of units) for all physical quantities.

8. [10 points]
A block of mass $M = 5.0\,\mathrm{kg}$, at rest on a horizontal frictionless table, is attached to a rigid support by a spring of constant $k = 510\,\mathrm{N/m}$ as shown in the figure below. A bullet of mass $m = 0.1\,\mathrm{kg}$ and speed $v = 51\,\mathrm{m/s}$ strikes and is embedded in the block. Assuming the compression of the spring is negligible until the bullet is embedded, determine the maximum compression of the spring.



9. [10 points]
How much electrical energy is converted into heat in 1.0 minute by a 20-Ω resistor carrying 0.5 A of current?

10. [10 points]
Determine the minimum energy to ionize a hydrogen atom in the n = 2 state? Note that the energy of the ground state (n = 1) is − 13.6 eV.

NAME:

2021 IUT Admission Test Answer Sheet for Contract-Based

1. [5 points]

Answer:

2. [5 points]

Answer:

3. [5 points]

Answer:

4. [5 points]

Answer:

5. [10 points]

Answer:

6. [20 points]

Answer:

7. [20 points]

Answer:

8. [10 points]

The total momentum of the system must be conserved before and after the collision.

$mv = (M+m)\,V$

Therefore, the speed of the block just after the collision

$V = \frac{m}{M+m}v = \frac{0.1\,\mathrm{kg}}{5.1\,\mathrm{kg}} \times 51\,\mathrm{m/s} = 1\,\mathrm{m/s}.$

Now due to the conservation of mechanical energy, the kinetic energy just after the collision should be same as the potential energy of the spring at the maximum compression. $\frac{1}{2}(M+m)\,V^2 = \frac{1}{2}kx^2$

Therefore,

$x = \sqrt{\frac{M+m}{k}}\,V = \sqrt{\frac{5.1\,\mathrm{kg}}{510\,\mathrm{N/m}}} \times 1\,\mathrm{m/s} = 0.1\,\mathrm{m}$

Answer: 0.1 m

9. [10 points]

The electrical energy converted into heat is given by $Q = I^2Rt$, where I is current, R is resistance, and t is time in seconds. Inserting the values in the problem, the heat is determined to be

$(0.5\,\mathrm{A})^2 \times 20\,\Omega \times 60\,\mathrm{sec} = 300\ \mathrm{J}$

Answer: 300 J

10. [10 points]

The energy level of a hydrogen atom is expressed as $E_n = \frac{-13.6\ \mathrm{eV}}{n^2}$, where n is an integer. So, the energy in the n = 2 state is $E_2 = \frac{-13.6\ \mathrm{eV}}{2^2} = -3.4\,\mathrm{eV}$.

Therefore, the minimum energy to ionize the hydrogen atom is $0 - E_2 = 3.4\ \mathrm{eV}$.

Answer: 3.4 eV