2021 IUT Admission Test(SBL)

Math Examination(TYPE A)

<**Multiple choice Types**> <u>There is only one correct answer for each question.</u> Mark your choice on the OMR answer sheet.

○ The points for each question are listed next to the question number.

○ No penalty point is applied to an incorrect answer.

1. [4 points]
When α, β are the solutions of $x^2-3x+1=0$, find $\alpha^2+\beta^2$.

① 1 ② 3 ③ 5 ④ 7 ⑤ 9

2. [4 points]
Simplify $\log_4 27\times\log_{25}8\times\log_3 5$.

① $\frac{1}{4}$ ② $\frac{3}{4}$ ③ $\frac{5}{4}$
④ $\frac{7}{4}$ ⑤ $\frac{9}{4}$

3. [4 points]
When $\sqrt{2\sqrt{2\sqrt[3]{2}}}=2^a$, find a.

① $\frac{7}{12}$ ② $\frac{2}{3}$ ③ $\frac{3}{4}$ ④ $\frac{5}{6}$ ⑤ $\frac{11}{12}$

4. [4 points]
Evaluate $\left(\frac{\sqrt{3}+i}{1+\sqrt{3}i}\right)^{100}$.

① $\frac{-\sqrt{3}+i}{2}$ ② $\frac{-1-\sqrt{3}i}{2}$
③ $\frac{\sqrt{3}+i}{2}$ ④ $\frac{1+\sqrt{3}i}{2}$ ⑤ $\frac{\sqrt{3}-i}{2}$

5. [4 points]
Evaluate $\sum_{n=1}^{20}(n^2-4n-5)$.

① 1910 ② 1930 ③ 1950
④ 1970 ⑤ 1990

6. [5 points]
Evaluate $\lim_{x\to\infty}\left(\sqrt{x^2+3x}-\sqrt{x^2-x}\right)$.

① 0 ② $\frac{1}{2}$ ③ 1 ④ $\frac{3}{2}$ ⑤ 2

7. [5 points]
When $f(x)=(x+1)^4(x^2+x+1)^7$, find $f'(0)$.

① 11 ② 12 ③ 13
④ 14 ⑤ 15

8. [5 points]
Evaluate $\int_1^{\sqrt{3}} x(x^2-1)^7dx$.

① $4\sqrt{3}$ ② 16 ③ $8\sqrt{3}$
④ 32 ⑤ $16\sqrt{3}$

9. [5 points]

Find the area of a triangle whose lengths of edges are 4, 5, 7.

① $4\sqrt{2}$ ② $4\sqrt{3}$ ③ 8
④ $4\sqrt{5}$ ⑤ $4\sqrt{6}$

10. [5 points]

When $A=\begin{pmatrix}1 & 1\\ 1 & -1\end{pmatrix}$, $B=\begin{pmatrix}2 & 1\\ 0 & 1\end{pmatrix}$ and $BAB^{-1}=\begin{pmatrix}a & b\\ c & d\end{pmatrix}$, find $a+b+c+d$.

① -1 ② $-\frac{1}{2}$ ③ 0
④ $\frac{1}{2}$ ⑤ 1

11. [5 points]

When $\sin\theta+\cos\theta=\frac{1}{2}$, find $\sin^4\theta+\cos^4\theta$.

① $\frac{21}{32}$ ② $\frac{23}{32}$ ③ $\frac{25}{32}$
④ $\frac{27}{32}$ ⑤ $\frac{29}{32}$

12. [5 points]

When a sequence $\{a_n\}_{n=1}^{\infty}$ satisfies

$$a_{n+1}=a_n+2n-1,\quad a_1=1$$

for every positive integer n, find a_{51}.

① 2501 ② 2506 ③ 2511
④ 2516 ⑤ 2521

13. [5 points]

When the function

$$f(x)=\begin{cases}\dfrac{x^2-ax+b}{x-1}, & x\neq 1\\ -2, & x=1\end{cases}$$

is continuous on the set of all real numbers, find $a+b$.

① 1 ② 3 ③ 5 ④ 7 ⑤ 9

14. [5 points]

Find the integer n satisfying $n\le \log_2(2^{10}\times 3^5)<n+1$.

① 15 ② 16 ③ 17
④ 18 ⑤ 19

15. [5 points]

Evaluate $\lim_{n\to\infty}\frac{1}{n}\sum_{k=1}^{n}\left(1+\frac{2k}{n}\right)^3$.

① 8 ② $\frac{17}{2}$ ③ 9
④ $\frac{19}{2}$ ⑤ 10

16. [6 points]

Find the sum of all solutions of

$$x^{\log_3 x-3}=81.$$

① $\frac{238}{3}$ ② $\frac{241}{3}$ ③ $\frac{244}{3}$
④ $\frac{247}{3}$ ⑤ $\frac{250}{3}$

17. [6 points]

Find the minimum value of the function

$$f(x) = \frac{x-1}{x^2+3} .$$

① $-\frac{3}{4}$ ② $-\frac{5}{8}$ ③ $-\frac{1}{2}$

④ $-\frac{3}{8}$ ⑤ $-\frac{1}{4}$

18. [6 points]

Let ℓ be the tangent line to the curve $y=x^3-2x$ at $x=1$. Find the area of the region enclosed by $y=x^3-2x$ and ℓ.

① $\frac{21}{4}$ ② $\frac{23}{4}$ ③ $\frac{25}{4}$

④ $\frac{27}{4}$ ⑤ $\frac{29}{4}$

19. [6 points]

When a polynomial $f(x)$ satisfies

$f(x)=x^3+x+\int_0^1 xf(x)dx,$

find $f(1)$.

① $\frac{34}{15}$ ② $\frac{37}{15}$ ③ $\frac{8}{3}$

④ $\frac{43}{15}$ ⑤ $\frac{46}{15}$

20. [6 points]

Let $f(x)$ be a differentiable function on the set of all real numbers with

$f(2)=1, \quad f'(2)=3.$

When $f(x)$ has the differentiable inverse function $g(x)$, find $\lim_{x\to1}\frac{g(x)-2}{x-1}$.

① $\frac{1}{3}$ ② $\frac{1}{2}$ ③ 1

④ 2 ⑤ 3

2021 IUT Admission Test Mathematics (SBL) Answers

Type A

1	2	3	4	5	6	7	8	9	10
④	⑤	④	②	②	⑤	①	②	⑤	③
11	12	13	14	15	16	17	18	19	20
②	①	④	③	⑤	③	③	④	⑤	①

2021 IUT Admission Test(SOCIE)

Math Examination(TYPE A)

< Multiple choice Types > There is only one correct answer for each question. Mark your choice on the OMR answer sheet.

○ The points for each question are listed next to the question number.

○ No penalty point is applied to an incorrect answer.

1. [4 points]

When $a=\dfrac{\sqrt{2-\sqrt{3}}}{\sqrt{2+\sqrt{3}}}$ and $b=\dfrac{\sqrt{2+\sqrt{3}}}{\sqrt{2-\sqrt{3}}}$, find $\sqrt{a}+\sqrt{b}$.

① $\sqrt{2}$ ② $\sqrt{6}$ ③ $\sqrt{8}$
④ $\sqrt{10}$ ⑤ $\sqrt{12}$

2. [4 points]

When $A=\begin{pmatrix} 5 & -3 \\ -2 & 1 \end{pmatrix}$, $B=\begin{pmatrix} 2 & -8 \\ 1 & -3 \end{pmatrix}$ and $A^{-1}+2B^{-1}=\begin{pmatrix} a & b \\ c & d \end{pmatrix}$, find $a+b+c+d$.

① -1 ② -3 ③ -5
④ -7 ⑤ -9

3. [4 points]

When $z=\dfrac{\sqrt{2}\,i}{1-i}$, find z^{2022}.

① 0 ② 1 ③ -1
④ i ⑤ $-i$

4. [4 points]

Find the sum of all solutions of $\cos(2x)+5\sin x+2=0$ for $0\le x\le 2\pi$.

① π ② $\dfrac{3}{2}\pi$ ③ 2π
④ $\dfrac{5}{2}\pi$ ⑤ 3π

5. [5 points]

When $1<a<b<a^3$ and $\log_a b+6\log_b a=5$, find $\log_a b+\log_b a$.

① $\dfrac{1}{2}$ ② $\dfrac{3}{2}$ ③ $\dfrac{5}{2}$
④ $\dfrac{7}{2}$ ⑤ $\dfrac{9}{2}$

6. [5 points]

When $\dfrac{\pi}{2}<\theta<\pi$ and $\sin\theta=\dfrac{1}{5}$, find $\cos\left(\theta+\dfrac{\pi}{6}\right)$.

① $-\dfrac{\sqrt{2}}{5}-\dfrac{1}{10}$ ② $-\dfrac{2\sqrt{2}}{5}-\dfrac{1}{10}$
③ $-\dfrac{3\sqrt{2}}{5}-\dfrac{1}{10}$ ④ $-\dfrac{4\sqrt{2}}{5}-\dfrac{1}{10}$
⑤ $-\sqrt{2}-\dfrac{1}{10}$

7. [5 points]

Let $\{a_n\}_{n=0}^{\infty}$ be defined by $a_0=3$, $a_n=2a_{n-1}$.

Find $\sum_{n=0}^{9} a_n$.

① 3057 ② 3060 ③ 3063
④ 3066 ⑤ 3069

8. [5 points]

When α, β, γ, δ are the solutions of $x^4-21x^2+3=0$, find $\dfrac{\alpha^2+\beta^2+\gamma^2+\delta^2}{\alpha\beta\gamma\delta}$.

① 12 ② 14 ③ 16
④ 18 ⑤ 20

9. [5 points]

When $f(x) = \dfrac{\sin x}{e^{2x}}$, find $f'\left(\dfrac{\pi}{4}\right)$.

① $-\dfrac{1}{\sqrt{2}}e^{-\frac{\pi}{2}}$ ② $-\dfrac{\sqrt{3}}{2}e^{-\frac{\pi}{2}}$ ③ 0

④ $\dfrac{1}{\sqrt{2}}e^{-\frac{\pi}{2}}$ ⑤ $\dfrac{\sqrt{3}}{2}e^{-\frac{\pi}{2}}$

10. [5 points]

Let M and m be the maximum and minimum values of

$$f(x) = 3x^4 - 4x^3 + a\ ,\quad 0 \le x \le 3\ .$$

When $M + m = 10$, find a.

① -60 ② -62 ③ -64

④ -66 ⑤ -68

11. [6 points]

Evaluate $\displaystyle\int_1^e x(\ln x)^2\,dx$.

① $\frac{1}{4}(e^2-1)$ ② $\frac{1}{4}(e^2-2)$ ③ $\frac{1}{4}(e^2-3)$

④ $\frac{1}{4}(e^2-4)$ ⑤ $\frac{1}{4}(e^2-5)$

12. [6 points]

Find the volume of the solid obtained by rotating the region enclosed by $y = tg\,x\ \left(0 \le x \le \dfrac{\pi}{4}\right)$, $y=0$ and $x = \dfrac{\pi}{4}$ about the x-axis, where $tgx = \dfrac{\sin x}{\cos x}$.

① $\pi - \dfrac{\pi^2}{4}$ ② $\pi - \dfrac{\pi^2}{8}$ ③ π

④ $\pi + \dfrac{\pi^2}{4}$ ⑤ $\pi + \dfrac{\pi^2}{8}$

13. [6 points]

Let f be a continuous function satisfying $x^2 + \displaystyle\int_a^x \frac{f(t)}{t^2}\,dt = 2x + 15$. When $a > 0$, find $f(a)$.

① -50 ② -100 ③ -150

④ -200 ⑤ -250

14. [6 points]

When $f(t) = \dfrac{10 + 4\sin t - \cos^2 t}{2 + \sin t}$, $(0 \le t \le 2\pi)$ has the minimum value at $t = t_0$, find $\sin t_0$.

① $\dfrac{-2+\sqrt{5}}{2}$ ② $\dfrac{2-\sqrt{5}}{2}$ ③ 0

④ $-2+\sqrt{5}$ ⑤ $2-\sqrt{5}$

2021 IUT Admission Test(SOCIE)

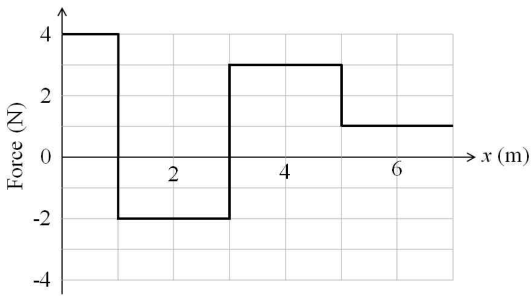
Physics Examination(A TYPE)

<Multiple choice Types> There is only one correct answer per each question. Mark your answer choice on the OMR answer sheet.

○ For each correct answer, you will get the points indicated next to each question number.

○ No penalty point is applied to an incorrect answer.

15. [3 point]
A force acting on a free particle varies with x, as shown in the figure below. Calculate the work done by the force as the particle moves from $x=0$ to $x=7$ m.

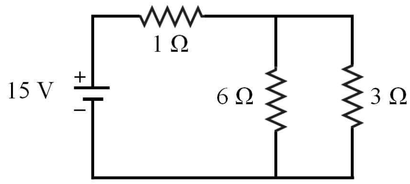


① 8 J ② 10 J ③ 12 J
④ 14 J ⑤ 16 J

16. [3 points]
A candle is imaged by a convex lens whose focal length is 20 cm. Find the distance between the image and the lens when the distance between the candle and the lens is 30 cm.

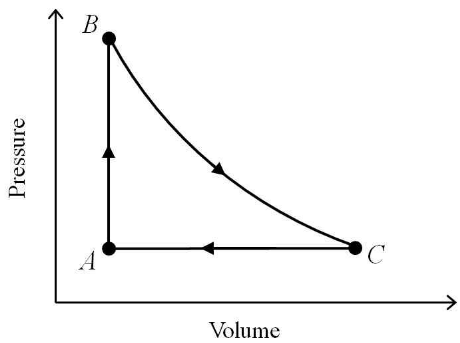
① 10 cm ② 20 cm ③ 30 cm
④ 50 cm ⑤ 60 cm

17. [3 point]
In the circuit shown below, what is the power dissipated in the 1-Ω resistor?



① 5 W ② 9 W ③ 16 W
④ 20 W ⑤ 25 W

18. [6 points]
As a gas is held within a closed chamber, it passes through the cycle shown in the figure. Determine the energy added to the gas as heat during constant-pressure process CA if the energy added as heat Q_{AB} during constant-volume process AB is 15.0 J, no energy is transferred as heat during adiabatic process BC, and the net work done during the cycle is 20.0 J.



① −35.0 J ② −5.0 J ③ 0.0 J
④ 5.0 J ⑤ 35.0 J

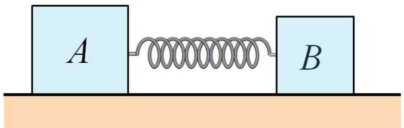
19. [6 point]

There are two identical conducting spheres with their center-to-center separation of 1.0 m. Initially, they are charged with charges $30.0\,\mu\mathrm{C}$ and $-10.0\,\mu\mathrm{C}$. The spheres are then connected by a thin conducting wire. When the wire is removed, what is the magnitude of the electrostatic force between the spheres? (Note that $1/(4\pi\epsilon_0) = 9\times10^9\ \mathrm{Nm^2/C^2}$.)

① 0.45 N ② 0.9 N ③ 1.8 N
④ 2.7 N ⑤ 3.6 N

20. [9 points]

Object A and object B are held together with a compressed spring between them on a frictionless floor as shown in the following figure. The mass of A is 2.00 times the mass of B. The spring constant of the spring is $k = 3000\ \mathrm{N/m}$ and it is compressed by 0.2 m. When they are released, the spring pushes them apart, and they then move in opposite directions, free of the spring. Assume that the spring has negligible mass and that all its stored energy is transferred to the objects. Once that transfer is complete, what is the kinetic energy of object B?



① 15 J ② 20 J ③ 30 J
④ 40 J ⑤ 45 J

2021 IUT Admission Test Mathematics (SOCIE) Answers

Type A

1	2	3	4	5	6	7	8	9	10
②	③	④	⑤	③	③	⑤	②	①	②
11	12	13	14						
①	①	④	④						