2021 IUT Admission Test (SOCIE, PreUniv. Type A)

Math Examination

<Essay Types> Applicants should write detailed solving process. If there is no solution, you will receive 0 points regardless of the correct answer.

○ The point for each question is indicated next to each question number.

1. [10 points]
When $a = \sqrt{3+\sqrt{5}}$ and $b = \sqrt{3-\sqrt{5}}$, find $a+b$.

2. [10 points]
When $\sin\alpha + \cos\alpha = \frac{1}{2}$, find $(\sin^2\alpha - \cos^2\alpha)^2$.

3. [10 points]
When $E = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$, $A = \begin{pmatrix} 3 & -4 \\ 1 & -3 \end{pmatrix}$, and
$$A^4 + A^3 - 6A^2 - 4A + 4E = \begin{pmatrix} a & b \\ c & d \end{pmatrix},$$
find $a+b+c+d$.

4. [20 points]
When α and β are solutions to $(2+\sqrt{3})^x + (2-\sqrt{3})^x = 4$, find $\alpha\beta$.

5. [20 points]
When $f(x) = x\sin(2x^2)$, find $f'\left(\sqrt{\frac{\pi}{12}}\right)$.

6. [30 points]
Find the area of the region enclosed by $y = x^4 + 3x^3 + 3$ and $y = x^4 + 2x^3 + 3x + 1$.

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(1) When $a=\sqrt{3+\sqrt{5}}$ and $b=\sqrt{3-\sqrt{5}}$, find $a+b$.

(SOL) : Since $ab=\sqrt{9-5}=2$,

$(a+b)^2=a^2+2ab+b^2=3+\sqrt{5}+2\cdot 2+3-\sqrt{5}=3+4+3=10$.

Since $a+b>0$, $a+b=\sqrt{10}$.

(2) When $\sin\alpha+\cos\alpha=\frac{1}{2}$, find $(\sin^2\alpha-\cos^2\alpha)^2$.

(SOL) : Since $\frac{1}{4}=(\sin\alpha+\cos\alpha)^2=1+2\sin\alpha\cos\alpha$, we get $\sin\alpha\cos\alpha=-\frac{3}{8}$.

Note that $(\sin^2\alpha-\cos^2\alpha)^2=(\sin^2\alpha+\cos^2\alpha)^2-4\sin^2\alpha\cos^2\alpha=1^2-4\left(-\frac{3}{8}\right)^2=\frac{7}{16}$. It follows that $(\sin^2\alpha-\cos^2\alpha)^2=\frac{7}{16}$

(3) When $E=\begin{pmatrix}1&0\\0&1\end{pmatrix}$, $A=\begin{pmatrix}3&-4\\1&-3\end{pmatrix}$, and $A^4+A^3-6A^2-4A+4E=\begin{pmatrix}a&b\\c&d\end{pmatrix}$, find $a+b+c+d$.

(SOL) : Note that $A^2=\begin{pmatrix}3&-4\\1&-3\end{pmatrix}\begin{pmatrix}3&-4\\1&-3\end{pmatrix}=\begin{pmatrix}5&0\\0&5\end{pmatrix}=5E$. Since

$A^4+A^3-6A^2-4A+4E=(5E)^2+(5E)A-6(5E)-4A+4E=A-E$, we get

$A^4+A^3-6A^2-4A+4E=A-E=\begin{pmatrix}3&-4\\1&-3\end{pmatrix}-\begin{pmatrix}1&0\\0&1\end{pmatrix}=\begin{pmatrix}2&-4\\1&-4\end{pmatrix}$. It follows that

$a+b+c+d=-5$.

(4) When α and β are solutions to $(2+\sqrt{3})^x+(2-\sqrt{3})^x=4$, find $\alpha\beta$.

(SOL) : Putting $t=(2+\sqrt{3}\,)^x$, then $(2-\sqrt{3}\,)^x=\frac{1}{(2+\sqrt{3}\,)^x}=\frac{1}{t}$, and the equation is reduced to $t+\frac{1}{t}=4$, which is $t^2-4t+1=0$. We get $t=2\pm\sqrt{3}$, which shows that $(2+\sqrt{3})^x=2+\sqrt{3}$, $(2+\sqrt{3})^x=2-\sqrt{3}=(2+\sqrt{3})^{-1}$. Hence, $x=1$ or $x=-1$. It follows that $\alpha\beta=-1$.

(5) When $f(x) = x\sin(2x^2)$, find $f'\left(\sqrt{\frac{\pi}{12}}\right)$.

(SOL) : Since $f'(x) = \sin(2x^2) + 4x^2\cos(2x^2)$, it follows that

$$f'\left(\sqrt{\frac{\pi}{12}}\right) = \sin\left(\frac{\pi}{6}\right) + \frac{\pi}{3}\cos\left(\frac{\pi}{6}\right) = \frac{3+\sqrt{3}\,\pi}{6}.$$

(6) Find the area of the region enclosed by $y = x^4 + 3x^3 + 3$ and $y = x^4 + 2x^3 + 3x + 1$.

(SOL) : Since $(x^4 + 3x^3 + 3) - (x^4 + 2x^3 + 3x + 1) = x^3 - 3x + 2 = (x-1)^2(x+2)$, two graphs meet at $x = -2$ and $x = 1$. Since $x^4 + 3x^3 + 3 \geq x^4 + 2x^3 + 3x + 1$ for $-2 \leq x \leq 1$, the area of region is

$$\int_{-2}^{1}[(x^4 + 3x^3 + 3) - (x^4 + 2x^3 + 3x + 1)]\,dx = \int_{-2}^{1}(x^3 - 3x + 2)\,dx$$

$$= \left[\frac{x^4}{4} - \frac{3}{2}x^2 + 2x\right]_{-2}^{1} = \frac{27}{4} .$$

2021 IUT Admission Test(SOCIE Pre U)

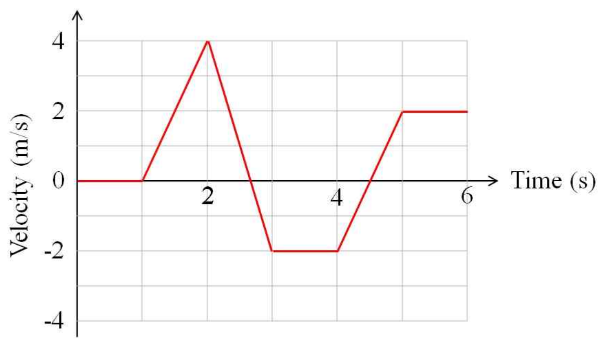
Physics Examination

<Essay Types> Applicants should write detailed solving process. If there is no solution, you will receive 0 points regardless of the correct answer.

○ The point for each question is indicated next to each question number.

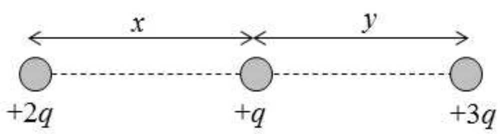
○ Be sure to use SI units (the international system of units) for all physical quantities.

1. [10 points]
Following figure gives the velocity of a particle moving on an x axis. Find the average acceleration in the time interval $t = 0$ to $t = 6$ s.



2. [10 points]
How much power is used by a 4-Ω resistor connected to a 6-V battery? (Ignore internal resistance of the battery.)

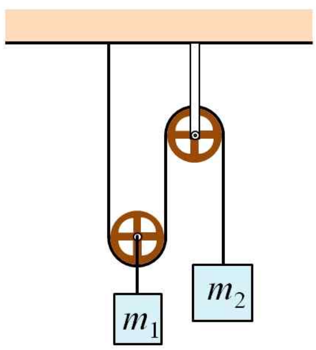
3. [10 points]
The figure below shows three point charges, all positive. If the net electric force on the center charge is zero, what is the value of $\frac{y}{x}$?



4. [20 points]
While riding in a car moving at a constant speed of 3.0 m/s on a horizontal road, you toss an egg directly upward with a initial relative speed of 10 m/s. When you catch the egg back in your hands at the same height, what is the horizontal displacement of the egg? (Assume that the magnitude of gravitational acceleration is 10 m/s^2 and ignore the effect of the air resistance)

5. [20 points]
An object of height 4 cm is placed 30 cm in front of a concave mirror whose focal length is 10 cm. Determine the height of the image.

6. [30 points]
Two blocks of masses $m_1 = m_2 = 1.0\,\text{kg}$ are connected by a cord to two frictionless pulleys with negligible masses as shown below. The left pulley is free to move and the right pulley is fixed to the ceiling. Determine the magnitude of acceleration of the block with mass m_2. (Assume that the magnitude of gravitational acceleration is 10 m/s^2.)



NAME:

2021 IUT Admission Test Answer Sheet for SOCIE Pre U Physics A

1. [10 points]

The average acceleration over a time interval $t_1 \le t \le t_2$ is given by $a_{avg} = \dfrac{v(t_2) - v(t_1)}{t_2 - t_1}$.

Therefore, $a_{avg} = \dfrac{2\,\mathrm{m/s} - 0\,\mathrm{m/s}}{6\,\mathrm{s} - 0\,\mathrm{s}} = \dfrac{2}{6}\,\mathrm{m/s^2} = \dfrac{1}{3}\,\mathrm{m/s^2}$

Answer: $\dfrac{1}{3}\,\mathrm{m/s^2}$

2. [10 points]

The power dissipated by a resistor is given by $P = IV = \dfrac{V^2}{R}$, where V is the voltage and R is the resistance. Therefore, $P = \dfrac{(6\,\mathrm{V})^2}{4\,\Omega} = 9\,\mathrm{W}$

Answer: 9 W

3. [10 points]

If the net electric force on the center charge is zero, the electrical repulsion by the +2q charge must balance the electrical repulsion by the +3q charge:

$k\dfrac{(2q)(q)}{x^2} = k\dfrac{(3q)(q)}{y^2}$, where $k = 1/(4\pi\epsilon_0)$.

Solving the equation yields $\dfrac{2}{x^2} = \dfrac{3}{y^2}$.

Consequently, $\dfrac{y}{x} = \sqrt{\dfrac{3}{2}}$

Answer: $\sqrt{\dfrac{3}{2}}$

4. [20 points]

The motion of the egg is same as the projectile motion under the gravitational force. The horizontal motion is that of a constant velocity and the vertical motion is that of a constant acceleration of $-g$. The vertical velocity is $v_y = v_0 - gt = 10\,\mathrm{m/s} - 10\,\mathrm{m/s^2} \times t$. Therefore it takes 2.0 s for you to catch the egg back. So the horizontal displacement of the egg

$\Delta x = v_x \Delta t = 3.0\,\mathrm{m/s} \times 2.0\,\mathrm{s} = 6.0\,\mathrm{m}$

Answer: **6.0 m**

5. [20 points]

The mirror equation is given by

$\dfrac{1}{o} + \dfrac{1}{i} = \dfrac{1}{f}$, where o and i are the position of the object and image, respectively, and f is the focal length. With o = 30 cm and f = 10 cm, the mirror equation is written as $\dfrac{1}{30\,\mathrm{cm}} + \dfrac{1}{i} = \dfrac{1}{10\,\mathrm{cm}}$,

which gives $i = 15\,\mathrm{cm}$. That is, the image is located 15 cm in front of the mirror.

The magnification is $m = |\dfrac{i}{o}| = \dfrac{1}{2}$. Therefore, the height of the image is $h_i = mh_o = \dfrac{1}{2} \times 4\,\mathrm{cm} = 2\,\mathrm{cm}$

Answer: **2 cm**

6. [30 points]

If the tension of the cord is T, the force on the block of mass m_1 is $F_1 = m_1 a_1 = 2T - m_1 g = 2T - 10\,\mathrm{N}$.

The force on the block of mass m_2 is

$F_2 = m_2 a_2 = m_2 g - T = 10\,\mathrm{N} - T$.

Since the distance moved by the block of mass m_2 is twice the distance moved by the block of mass m_1, $a_2 = 2a_1$. Inserting all the given values, we can obtain $a_2 = 4.0\,\mathrm{m/s^2}$

Answer: **4.0 m/s²**