

2023 IUT 2nd Admission Test(SBL)
Math Examination(TYPE A)

< Multiple choice Types > There is only one correct answer for each question. Mark your choice on the OMR answer sheet.

- The points for each question are listed next to the question number.
- You can use the right side of each page for your memo.

1. [3 points]

Compute $\sqrt{7+\sqrt{48}} + \sqrt{7-\sqrt{48}}$.

- ① 2 ② 4 ③ 6 ④ 8 ⑤ 10

2. [3 points]

When $a + \frac{2}{a} = 4$, find $a^3 + \frac{8}{a^3}$.

- ① 10 ② 20 ③ 30 ④ 40 ⑤ 50

3. [3 points]

Compute $\sum_{n=2}^9 \frac{1}{n^2-1}$.

- ① $\frac{3}{5}$ ② $\frac{8}{15}$ ③ $\frac{13}{27}$ ④ $\frac{17}{32}$ ⑤ $\frac{29}{45}$

4. [3 points]

Compute $\log_2 3 + (\log_2 3 \times \log_3 4) + \log_{\frac{1}{2}} 3$.

- ① 2 ② 4 ③ 6 ④ 8 ⑤ 10

5. [3 points]

When $4^x = \sqrt{5}$, $8^y = \frac{1}{5}$, find $\frac{1}{x} - \frac{1}{y}$.

- ① $\log_5 2$ ② $3 \log_5 2$ ③ $5 \log_5 2$
④ $7 \log_5 2$ ⑤ $9 \log_5 2$

6. [3 points]

When $A = \begin{pmatrix} 1 & 2 \\ -2 & -3 \end{pmatrix}$, $B = \begin{pmatrix} 4 & -1 \\ -3 & 2 \end{pmatrix}$ and

$A^{-1}BA = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$, find $a+b+c+d$.

- ① 2 ② 4 ③ 6 ④ 8 ⑤ 10

7. [3 points]

When $A = \begin{pmatrix} 2 & 3 \\ 7 & 4 \end{pmatrix}$, $B = \begin{pmatrix} 6 & 8 \\ 11 & 2 \end{pmatrix}$ and

$A(A^{-1} - 2B^{-1})B = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$, find $a+b+c+d$.

- ① -3 ② -5 ③ -7 ④ -9 ⑤ -11

8. [3 points]

Find the sum of all solutions of

$$2^{\log_3 x} + 2^{3-\log_3 x} = 6.$$

- ① 12 ② 15 ③ 18 ④ 21 ⑤ 24

9. [3 points]

When $a = \log_{\sqrt{3}}(\sqrt{5}+2)$, find $3^a + \frac{1}{3^a}$.

- ① 10 ② 12 ③ 14 ④ 16 ⑤ 18

10. [3 points]

Find the minimum value of

$$f(x) = (\log_3 x)^2 + 2\log_{\frac{1}{3}}(3x), \quad x > 0.$$

- ① -1 ② -3 ③ -5 ④ -7 ⑤ -9

11. [3 points]

When $\omega^3 = 1$ and $\omega \neq 1$, find

$$7\omega^{24} + 3\omega^{20} + 3\omega^{16} + \omega^{15}.$$

- ① 1 ② 3 ③ 5 ④ 7 ⑤ 9

12. [3 points]

When $x^2 - 3x + 1 = 0$, compute $x^3 + \frac{1}{x^3}$.

- ① 12 ② 15 ③ 18 ④ 21 ⑤ 24

13. [3 points]

$$\text{Compute } \left(\frac{-1 + \sqrt{3}i}{\sqrt{3} + i} \right)^{111}.$$

- ① 0 ② 1 ③ -1 ④ i ⑤ $-i$

14. [3 points]

Compute

$$\frac{1}{1-i} + \frac{1+i}{(1-i)^2} + \frac{(1+i)^2}{(1-i)^3} + \dots + \frac{(1+i)^9}{(1-i)^{10}}.$$

- ① 0 ② 1 ③ -1 ④ i ⑤ $-i$

15. [3 points]

When α , β and γ are solutions of

$$x^3 - 2x^2 + 3x - 1 = 0, \text{ find } \alpha^2\beta^2 + \beta^2\gamma^2 + \gamma^2\alpha^2.$$

- ① 1 ② 3 ③ 5 ④ 7 ⑤ 9

16. [3 points]

Find $\cos \frac{\pi}{8}$.

① $\frac{\sqrt{2+\sqrt{2}}}{2}$ ② $\frac{\sqrt{2-\sqrt{2}}}{2}$ ③ $\frac{\sqrt{2+\sqrt{2}}}{3}$

④ $\frac{\sqrt{2-\sqrt{2}}}{3}$ ⑤ $\frac{\sqrt{3-\sqrt{2}}}{6}$

17. [3 points]

Find the sum of all solutions of $\cos 2x + \sin x = 0$ for $0 \leq x \leq 2\pi$.

- ① $\frac{5\pi}{2}$ ② 3π ③ $\frac{7\pi}{2}$ ④ 4π ⑤ $\frac{9\pi}{2}$

18. [3 points]

When α and β are the solutions of $x^2 - 20x + 4 = 0$ with $0 < \alpha < \beta$,

$$\text{find } \frac{1}{\alpha\sqrt{\beta}} - \frac{1}{\beta\sqrt{\alpha}}.$$

- ① 1 ② 3 ③ 5 ④ 7 ⑤ 9

19. [3 points]

Find the sum of all solutions of

$$\sqrt{3} \sin x + \cos x = \sqrt{2} \text{ for } 0 \leq x \leq 2\pi.$$

- ① $\frac{\pi}{3}$ ② $\frac{2\pi}{3}$ ③ π ④ $\frac{4\pi}{3}$ ⑤ $\frac{5\pi}{3}$

20. [3 points]

$$\text{Find } \lim_{x \rightarrow 0} \frac{3x^3}{(1 - \cos x) \sin 5x}.$$

- ① $\frac{2}{5}$ ② $\frac{4}{5}$ ③ $\frac{6}{5}$ ④ $\frac{8}{5}$ ⑤ 2

21. [4 points]

Find the minimum value of

$$f(x) = 6x^4 - 4x^3 + 36x^2 - 36x + \frac{1}{8}.$$

- ① -1 ② -3 ③ -5 ④ -7 ⑤ -9

22. [4 points]

When $y = ax + b$ is the tangent line to $f(x) = -x^3 - 3x^2 + x + 5$ at $x = -1$, find $a + b$.

- ① 2 ② 4 ③ 6 ④ 8 ⑤ 10

23. [4 points]

When M and m are the maximum and minimum values of $f(x) = x^3 - 6x^2 + 9x + 2$ for $-1 \leq x \leq 2$, find $M + m$.

- ① -2 ② -4 ③ -6 ④ -8 ⑤ -10

24. [4 points]

When $y = -2x + a$ is tangent to $f(x) = x^2 + 4x + 5$, find a .

- ① -2 ② -4 ③ -6 ④ -8 ⑤ -10

25. [4 points]

When $f(x) = (x^3 - 3x - 1)^4$, find $f'(2)$.

- ① 9 ② 18 ③ 27 ④ 36 ⑤ 45

26. [4 points]

Compute $\int_0^1 (3x - 2)^5 dx$.

- ① $-\frac{1}{2}$ ② $-\frac{3}{2}$ ③ $-\frac{5}{2}$ ④ $-\frac{7}{2}$ ⑤ $-\frac{9}{2}$

27. [4 points]

When $f(x) = \frac{x}{2 + \sqrt{x}}$, find $f'(1)$.

- ① $\frac{5}{6}$ ② $\frac{5}{12}$ ③ $\frac{5}{18}$ ④ $\frac{5}{24}$ ⑤ $\frac{1}{6}$

28. [4 points]

Compute $\int_0^2 x \sqrt{2x^2 + 1} dx$.

- ① $\frac{11}{3}$ ② $\frac{13}{3}$ ③ 5 ④ $\frac{17}{3}$ ⑤ $\frac{19}{3}$

29. [4 points]

Find the area of the region enclosed by two curves $y = x^3 - 3x^2 + 3x + 1$ and $y = x^3 - x^2 + 3x - 1$.

- ① $\frac{2}{3}$ ② $\frac{4}{3}$ ③ 2 ④ $\frac{8}{3}$ ⑤ $\frac{10}{3}$

30. [4 points]

When a differentiable function $f : \mathbb{R} \rightarrow \mathbb{R}$

satisfies $\int_0^x (f(t) - t^2) dt = x(x^2 - 1)^2$,

find $f'(1)$.

- ① 2 ② 4 ③ 6 ④ 8 ⑤ 10

2023 IUT 2nd SBL Answer Sheets

[TypeA]

No.	1	2	3	4	5	6	7	8	9	10
Ans.	②	④	⑤	①	④	①	②	①	⑤	②
No.	11	12	13	14	15	16	17	18	19	20
Ans.	③	③	⑤	④	③	①	③	①	②	③
No.	21	22	23	24	25	26	27	28	29	30
Ans.	⑤	⑤	④	②	④	④	③	②	④	⑤

Math Examination(TYPE A)

< Multiple choice Types > There is only one correct answer for each question. Mark your choice on the OMR answer sheet.

- The points for each question are listed next to the question number.
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1. [2 points]

Find $\frac{1}{\sqrt{5}+\sqrt{3}} - \frac{1}{\sqrt{5}-\sqrt{3}}$.

- ① $-\sqrt{2}$ ② $-\sqrt{3}$ ③ $-\sqrt{5}$
 ④ $-2\sqrt{2}$ ⑤ $-\sqrt{15}$

2. [2 points]

Find $\sqrt{3+2\sqrt{2}} - \sqrt{3-2\sqrt{2}}$.

- ① 2 ② 4 ③ 6 ④ 8 ⑤ 10

3. [2 points]

Simplify $\frac{\sqrt[6]{2} \times \sqrt[4]{6}}{\sqrt[3]{4}}$.

- ① $\sqrt{\frac{3}{2}}$ ② $\sqrt[3]{\frac{3}{2}}$ ③ $\sqrt[4]{\frac{3}{2}}$
 ④ $\sqrt[6]{\frac{3}{2}}$ ⑤ $\sqrt[12]{\frac{3}{2}}$

4. [2 points]

When $\sqrt{x} + \frac{1}{\sqrt{x}} = 2\sqrt{3}$, find $x^2 + \frac{1}{x^2}$.

- ① 90 ② 92 ③ 94 ④ 96 ⑤ 98

5. [2 points]

Simplify $\frac{1+\sqrt{3}+\sqrt{2}}{1+\sqrt{3}-\sqrt{2}}$.

- ① $\sqrt{2}$ ② $\sqrt{3}$ ③ $\sqrt{6}$
 ④ $\sqrt{2}+\sqrt{3}$ ⑤ $-\sqrt{2}+\sqrt{3}$

6. [2 points]

When α and β are solutions of $x^2+2x+5=0$, find $\alpha^3\beta+\alpha\beta^3$.

- ① -22 ② -24 ③ -26 ④ -28 ⑤ -30

7. [2 points]

When α, β, γ are the solutions of

$2x^3+3x^2+4x+5=0$, find $\frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\gamma}$.

- ① $-\frac{2}{5}$ ② $-\frac{4}{5}$ ③ $-\frac{6}{5}$ ④ $-\frac{8}{5}$ ⑤ -2

8. [2 points]

When $\frac{1}{x} + \frac{1}{y} = \frac{3}{2}$ and $2^x = 3^y$, find 8^x .

- ① 30 ② 32 ③ 34 ④ 36 ⑤ 38

9. [2 points]

When $3^x - 3^y = 6$, $3^{x+y} = 27$, find $2x - 3y$.

- ① 0 ② $\frac{1}{2}$ ③ 1 ④ $\frac{3}{2}$ ⑤ 2

10. [2 points]

Find the largest real number a for which the system of equations $x + y = 2a + 2$ and $xy = 3a^2 + 1$ has a real solution.

- ① 1 ② 3 ③ 5 ④ 7 ⑤ 9

11. [2 points]

When $x = \log_2(2 + \sqrt{3})$, find $4^x + 4^{-x}$.

- ① 10 ② 12 ③ 14 ④ 16 ⑤ 18

12. [2 points]

When $A = \log_3 5$, express $\log_{15} 5$ in terms of A .

- ① $\frac{A}{A+1}$ ② $\frac{A-1}{A+1}$ ③ $\frac{A+1}{A}$
④ $\frac{A+1}{A-1}$ ⑤ $\frac{A+1}{A^2+1}$

13. [2 points]

Find the sum of all solutions of $\log_2 x \times \log_{\frac{1}{4}} x = -2$.

- ① $\frac{11}{4}$ ② $\frac{13}{4}$ ③ $\frac{15}{4}$ ④ $\frac{17}{4}$ ⑤ $\frac{19}{4}$

14. [2 points]

Find the sum of all solutions of

$$3^{x+1} + 5 \cdot 3^{-x} - 9 = 0.$$

- ① $\log_3 5$ ② $\log_3 5 + 1$ ③ $\log_3 5 + 2$
④ $\log_3 5 - 1$ ⑤ $\log_3 5 - 2$

15. [2 points]

When $\omega^3 = 1$ and $\omega \neq 1$, find $\omega^{2023} + \omega^{-2024}$.

- ① -2ω ② $-\omega$ ③ 0 ④ ω ⑤ 2ω

16. [2 points]

When $z = \sqrt{2} + i$ is a solution of $z^4 + az^2 + b = 0$ for some real numbers a and b , find b .

- ① 1 ② 3 ③ 5 ④ 7 ⑤ 9

17. [2 points]

When α, β, γ are the solutions of

$$x^3 - 2x^2 + 3x - 4 = 0, \text{ find } \frac{\beta + \gamma}{\alpha} + \frac{\alpha + \gamma}{\beta} + \frac{\alpha + \beta}{\gamma}.$$

- ① $-\frac{1}{2}$ ② $-\frac{3}{2}$ ③ $-\frac{5}{2}$ ④ $-\frac{7}{2}$ ⑤ $-\frac{9}{2}$

18. [2 points]

When $\cos \alpha = \frac{\sqrt{2}}{3}$, $\sin \beta = \frac{1}{3}$ for

$0 < \alpha < \frac{\pi}{2} < \beta < \pi$, find $\cos(\alpha + \beta)$.

- ① $-\frac{\sqrt{7}}{9}$ ② $-\frac{2 + \sqrt{7}}{9}$ ③ $-\frac{4 + \sqrt{7}}{9}$
④ $-\frac{6 + \sqrt{7}}{9}$ ⑤ $-\frac{8 + \sqrt{7}}{9}$

19. [2 points]

When $\operatorname{tg} \alpha$ and $\operatorname{tg} \beta$ ($-\frac{\pi}{2} < \alpha < 0 < \beta < \frac{\pi}{2}$) are the solutions of $x^2 - 4x - 7 = 0$, find $\cos(\alpha + \beta)$, where $\operatorname{tg} \theta = \frac{\sin \theta}{\cos \theta}$.

- ① $\frac{\sqrt{5}}{3}$ ② $\frac{\sqrt{5}}{5}$ ③ $\frac{2\sqrt{5}}{5}$
④ $\frac{\sqrt{5}}{7}$ ⑤ $\frac{2\sqrt{5}}{7}$

20. [2 points]

When M and m are the maximum and minimum values of $f(x) = 4\sin\left(x + \frac{5\pi}{3}\right) + 2\sqrt{3}\cos x + 1$, find $M + m$.

- ① $\frac{1}{2}$ ② 1 ③ $\frac{3}{2}$ ④ 2 ⑤ $\frac{5}{2}$

21. [2 points]

Find the sum of all solutions of $\log_{\sqrt{2}}(x+1) - \log_2(14x+2) = -2$.

- ① $\frac{1}{2}$ ② $\frac{3}{2}$ ③ $\frac{5}{2}$ ④ $\frac{7}{2}$ ⑤ $\frac{9}{2}$

22. [2 points]

When $A = \begin{pmatrix} 1 & 1 \\ 4 & 3 \end{pmatrix}$, $B = \begin{pmatrix} -1 & 2 \\ -3 & -2 \end{pmatrix}$ and

$A^{-1}BA = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$, find $a + b + c + d$.

- ① 3 ② 6 ③ 9 ④ 12 ⑤ 15

23. [2 points]

When $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ satisfies $A \begin{pmatrix} 1 \\ 2 \end{pmatrix} = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$ and $A^2 \begin{pmatrix} 1 \\ 2 \end{pmatrix} = \begin{pmatrix} -2 \\ 8 \end{pmatrix}$, find $a + b + c + d$.

- ① -4 ② -2 ③ 0 ④ 2 ⑤ 4

24. [2 points]

When $y = x + a$ is tangent to $f(x) = x^4 - 3x + 5$, find a .

- ① 2 ② 4 ③ 6 ④ 8 ⑤ 10

25. [2 points]

When $x^{2023} + 1 = A(x)(x-1)^2 + ax + b$ for some polynomial $A(x)$, find b .

- ① -2015 ② -2017 ③ -2019
④ -2021 ⑤ -2023

26. [3 points]

When $A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$ and $B = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ satisfy $ABA = A$, find $a + b + c + d$

- ① -4 ② -2 ③ 0 ④ 2 ⑤ 4

27. [3 points]

Find $\lim_{x \rightarrow 0} \frac{9x + \sin 3x}{4x + \sin 2x}$.

- ① 2 ② 4 ③ 6 ④ 8 ⑤ 10

28. [3 points]

Find $\lim_{x \rightarrow 0} \frac{3 - \sqrt{x^2 + 9}}{x \sin x}$.

- ① $-\frac{1}{2}$ ② $-\frac{1}{3}$ ③ $-\frac{1}{4}$ ④ $-\frac{1}{5}$ ⑤ $-\frac{1}{6}$

29. [3 points]

When M and m are the maximum and minimum values of $f(x) = x^3 - 6x^2 + 9x + 3$ ($0 \leq x \leq 3$), find $M + m$.

- ① 2 ② 4 ③ 6 ④ 8 ⑤ 10

30. [3 points]

Find the maximum value of $f(x) = -3x^4 + 4x^3 - 6x^2 + 12x + 5$.

- ① 10 ② 12 ③ 14 ④ 16 ⑤ 18

31. [3 points]

When $y = ax + b$ is the tangent line to $f(x) = \sqrt{x^2 + 2x + 8}$ at $x = 2$, find $a + b$.

- ① $\frac{11}{4}$ ② $\frac{13}{4}$ ③ $\frac{15}{4}$ ④ $\frac{17}{4}$ ⑤ $\frac{19}{4}$

32. [3 points]

When $f(x) = \frac{x}{\sqrt{x^2 + 5}}$, find $f'(2)$.

- ① $\frac{1}{27}$ ② $\frac{1}{9}$ ③ $\frac{5}{27}$ ④ $\frac{7}{27}$ ⑤ $\frac{1}{3}$

33. [3 points]

Find $\int_{-1}^2 (x^3 + x^2 - 1) dx$.

- ① $\frac{11}{4}$ ② $\frac{13}{4}$ ③ $\frac{15}{4}$ ④ $\frac{17}{4}$ ⑤ $\frac{19}{4}$

34. [3 points]

Find $\int_0^2 \frac{x}{(\sqrt{2x^2 + 1})^3} dx$.

- ① $\frac{1}{2}$ ② $\frac{1}{3}$ ③ $\frac{1}{4}$ ④ $\frac{1}{6}$ ⑤ $\frac{1}{9}$

35. [3 points]

Find $\int_0^2 (x^2 - 2x + 1)^5 dx$.

- ① $\frac{2}{11}$ ② $\frac{4}{11}$ ③ $\frac{6}{11}$ ④ $\frac{8}{11}$ ⑤ $\frac{10}{11}$

36. [4 points]

When $\{a_n\}_{n=1}^{\infty}$, $\{b_n\}_{n=1}^{\infty}$ satisfy $a_1 = 7$, $b_1 = 10$, and $a_{n+1} = 3b_n + 1$, $b_{n+1} = 3a_n - 5$,

find $\lim_{n \rightarrow \infty} \frac{a_n + b_n}{3^n}$.

- ① 1 ② 3 ③ 5 ④ 7 ⑤ 9

37. [4 points]

When a differentiable function $f: \mathbb{R} \rightarrow \mathbb{R}$ satisfies $\int_0^x f(t^2) dt = x(x^2 + 1)^3$, find $f'(1)$.

- ① 12 ② 24 ③ 36 ④ 48 ⑤ 60

38. [4 points]

Find $\lim_{n \rightarrow \infty} \sum_{k=1}^n \frac{3k^2 + 2n^2}{n^3}$.

- ① 1 ② 3 ③ 5 ④ 7 ⑤ 9

39. [4 points]

When $f(x) = x^3 + 3 - \int_0^x 2f'(t) dt$, find $f(3)$.

- ① 12 ② 14 ③ 16 ④ 18 ⑤ 20

40. [4 points]

Find the area of the region enclosed by $y = x^2 + 3x - 3$ and $y = 2x - 1$.

- ① $\frac{1}{2}$ ② $\frac{3}{2}$ ③ $\frac{5}{2}$ ④ $\frac{7}{2}$ ⑤ $\frac{9}{2}$

2023 IUT 2nd SBL Scholar Answer Sheets

[TypeA]

No.	1	2	3	4	5	6	7	8	9	10
Ans.	②	①	③	⑤	④	⑤	②	④	③	①
No.	11	12	13	14	15	16	17	18	19	20
Ans.	③	①	④	④	⑤	⑤	②	③	③	④
No.	21	22	23	24	25	26	27	28	29	30
Ans.	②	④	④	①	④	③	①	⑤	⑤	②
No.	31	32	33	34	35	36	37	38	39	40
Ans.	②	③	③	②	①	③	⑤	②	①	⑤

Math Examination(TYPE A)

< Multiple choice Types > There is only one correct answer for each question. Mark your choice on the OMR answer sheet.

- The points for each question are listed next to the question number.
- You can use the right side of each page for your memo.

1. [2 points]

Simplify $(\sqrt{2} + \sqrt{8} + \sqrt{9})(\sqrt{2} + \sqrt{8} - \sqrt{9})$.

- ① 7 ② 8 ③ 9 ④ 10 ⑤ 11

2. [2 points]

Simplify $\frac{\log_2 3 + \log_4 27}{\log_4 36 - \log_2 18}$.

- ① $-\frac{5}{2}$ ② $-\frac{1}{2}$ ③ 0 ④ $\frac{1}{2}$ ⑤ $\frac{5}{2}$

3. [2 points]

When $\sin\theta + \cos\theta = \frac{\sqrt{2}}{2}$, find $\sin^3\theta + \cos^3\theta$.

- ① $\frac{\sqrt{2}}{8}$ ② $\frac{\sqrt{2}}{4}$ ③ $\frac{3\sqrt{2}}{8}$
 ④ $\frac{\sqrt{2}}{2}$ ⑤ $\frac{5\sqrt{2}}{8}$

4. [2 points]

When $\omega^3 = 1$ and $\omega \neq 1$,
simplify $1 + \omega + \omega^2 + \dots + \omega^{100}$.

- ① $-\omega^2$ ② $-\omega$ ③ 1 ④ ω ⑤ ω^2

5. [2 points]

Evaluate $\sum_{n=1}^{20} \ln\left(\frac{n+1}{n}\right)$.

- ① $\ln 19$ ② $\ln 21$ ③ $\ln 23$ ④ $\ln 25$ ⑤ $\ln 27$

6. [2 points]

When $e^{2x} = 2$, evaluate $\frac{e^{3x} - e^{-3x}}{e^x + e^{-x}}$.

- ① $\frac{7}{6}$ ② $\frac{4}{3}$ ③ $\frac{3}{2}$ ④ $\frac{5}{6}$ ⑤ $\frac{11}{6}$

7. [3 points]

Evaluate $\lim_{x \rightarrow \infty} (\sqrt{x^2 + 3x} - x)$.

- ① 1 ② $\frac{3}{2}$ ③ 2 ④ $\frac{5}{2}$ ⑤ 3

8. [3 points]

When α, β are solutions of $x^{\ln x} = ex^3$, find $\alpha\beta$.

- ① 1 ② e ③ e^2 ④ e^3 ⑤ e^4

9. [3 points]

For a triangle $\triangle ABC$ with $\overline{AB}=5$, $\overline{AC}=4$, $\overline{BC}=7$, find $\cos(\angle A)$.

- ① $-\frac{1}{5}$ ② $-\frac{\sqrt{2}}{5}$ ③ $-\frac{2}{5}$
④ $-\frac{2\sqrt{2}}{5}$ ⑤ $-\frac{4}{5}$

10. [3 points]

Find the maximum of $f(x) = \sqrt{2}\cos x + \sin x$.

- ① 1 ② $\sqrt{2}$ ③ $\sqrt{3}$ ④ 2 ⑤ $\sqrt{5}$

11. [3 points]

When $x = 1 + t^3$, $y = t^2 + 3t$, find $\frac{dy}{dx}$ at $t = 1$.

- ① $\frac{1}{3}$ ② $\frac{2}{3}$ ③ 1 ④ $\frac{4}{3}$ ⑤ $\frac{5}{3}$

12. [3 points]

Compute $\int_0^\pi \sin x \cos 2x dx$.

- ① $-\frac{2}{3}$ ② $-\frac{1}{3}$ ③ 0 ④ $\frac{1}{3}$ ⑤ $\frac{2}{3}$

13. [4 points]

When a sequence $\{a_n\}_{n=1}^\infty$ satisfies $a_1 = 2$ and

$$a_{n+1} = \frac{n}{n+1}a_n, \quad n = 1, 2, 3, \dots,$$

find a_{50} .

- ① $\frac{1}{50}$ ② $\frac{1}{25}$ ③ $\frac{3}{50}$ ④ $\frac{2}{25}$ ⑤ $\frac{1}{10}$

14. [4 points]

Find the radius of a circle passing through three points $(5, 3)$, $(-2, 4)$, $(-3, -3)$.

- ① 2 ② $2\sqrt{2}$ ③ 4 ④ $3\sqrt{2}$ ⑤ 5

15. [4 points]

When $A = \begin{pmatrix} 3 & -1 \\ 7 & -2 \end{pmatrix}$ and $A + A^2 + \dots + A^{20} = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$,

find $a + b + c + d$.

- ① 3 ② 6 ③ 9 ④ 12 ⑤ 15

16. [4 points]

Find the distance between two lines

$$3x + 4y + 1 = 0, \quad 3x + 4y - 9 = 0.$$

- ① 1 ② $\sqrt{2}$ ③ $\sqrt{3}$ ④ 2 ⑤ $\sqrt{5}$

17. [4 points]

When θ is the angle between two lines

$$y = \frac{1}{2}x - 1 \text{ and } y = \frac{1}{3}x + 2, \text{ find } \sin\theta.$$

- ① $\frac{1}{10}$ ② $\frac{\sqrt{2}}{10}$ ③ $\frac{1}{5}$
④ $\frac{\sqrt{2}}{5}$ ⑤ $\frac{3\sqrt{2}}{10}$

18. [5 points]

When a line $y = mx + 2$ is tangent to the curve

$$y = x^3 + 6x, \text{ find the constant } m.$$

- ① 1 ② 3 ③ 5 ④ 7 ⑤ 9

19. [5 points]

Let f be given by

$$f(x) = \begin{cases} \frac{a - \cos x}{x^2} & (x \neq 0), \\ b & (x = 0) \end{cases}$$

for constants a, b . When f is continuous over real numbers, find $a + b$.

- ① $\frac{1}{2}$ ② 1 ③ $\frac{3}{2}$ ④ 2 ⑤ $\frac{5}{2}$

20. [5 points]

$$\text{Evaluate } \lim_{n \rightarrow \infty} \sum_{k=1}^n k \ln \sqrt{\frac{n+k}{n}}.$$

- ① 1 ② $\frac{1}{2}$ ③ $\frac{1}{3}$ ④ $\frac{1}{4}$ ⑤ $\frac{1}{5}$

21. [5 points]

Find the volume of the solid obtained by rotating the region bounded by the x -axis and

$$y = \cos x - 1 \quad (0 \leq x \leq 2\pi)$$

about the x -axis.

- ① π^2 ② $2\pi^2$ ③ $3\pi^2$ ④ $4\pi^2$ ⑤ $5\pi^2$

Answers to 2023 IUT SOCIE Admission Test (Contract)

- Type A

1	2	3	4	5	6	7
③	①	⑤	①	②	①	②
8	9	10	11	12	13	14
④	①	③	⑤	①	②	⑤
15	16	17	18	19	20	21
④	④	②	⑤	③	④	③

2023 IUT Admission Test(SOCIE)
Physics Examination(A TYPE)

<Multiple choice Types> There is only one correct answer per each question. Mark your answer choice on the OMR answer sheet.

- For each correct answer, you will get the points indicated next to each question number.
- No penalty point is applied to an incorrect answer.

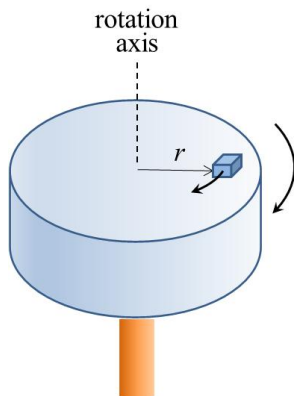
1. [3 points]

Which of the following physical quantities is not a vector quantity?

- ① velocity ② force ③ work
- ④ momentum ⑤ electric field

2. [4 points]

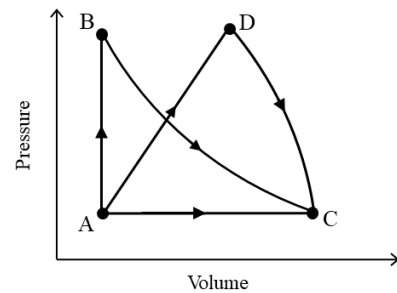
As shown in the figure below, a large circular disk placed horizontally is rotating with an angular velocity of $\omega = 2.0 \text{ rad/s}$ around a vertical rotation axis. An object with a coefficient of static friction $\mu = 0.60$ with the disk is placed on the disk to make it rotate together with the disk. How far can the object be placed from the rotation axis of the disk? (However, it is assumed that the magnitude of the gravitational acceleration is $g = 10 \text{ m/s}^2$.)



- ① 0.6 m ② 0.8 m ③ 1.0 m
- ④ 1.5 m ⑤ 1.8 m

3. [3 points]

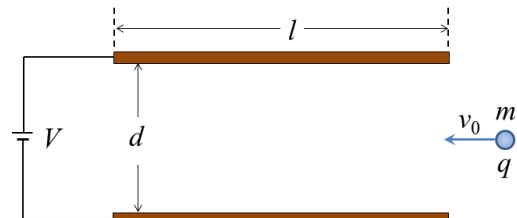
The figure below shows the various state change paths of an ideal gas in a pressure–volume graph. Which of the following processes can be considered as an isothermal process?



- ① A→B ② A→C ③ A→D
- ④ B→C ⑤ D→C

4. [5 points]

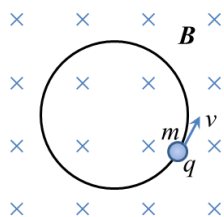
A voltage V is applied between two parallel plates with distance d and length l as shown in the figure below. A particle of mass m and charge q is incident into the parallel plates with an initial velocity v_0 parallel to plates. It is assumed that a uniform electric field is created between the parallel plates. Assuming that the particle does not collide with plates, what is the speed of the particle as it exits the plates? (However, ignore the effect of gravity.)



- ① $v_0 + \frac{qlV}{mdv_0}$ ② $\sqrt{v_0^2 + \left(\frac{qlV}{mdv_0}\right)^2}$
- ③ $v_0 + \frac{qlV}{2md}v_0$ ④ $\sqrt{v_0^2 + \left(\frac{qlV}{md}v_0\right)^2}$
- ⑤ $\sqrt{v_0^2 + \left(\frac{qlV}{2md}v_0\right)^2}$

5. [3 points]

As shown in the figure below, when a particle of mass m and charge q is incident perpendicularly to a uniform magnetic field B with velocity v , the particle moves in a circle. What is the radius of circular motion?



- ① $\frac{Bv}{2mq}$ ② $\frac{mq}{2Bv}$ ③ $\frac{Bv}{mq}$
 ④ $\frac{mq}{Bv}$ ⑤ $\frac{mv}{Bq}$

6. [3 points]

Considering the direction of wave propagation and the direction of vibration of the medium, which of the following waves has a different property from the rest?

- ① sound wave ② microwave ③ x-ray
 ④ string wave ⑤ water wave

7. [3 points]

Which of the following statements about the image of a mirror is correct?

- ① The image by a flat mirror is always an upright real image.
 ② The virtual image produced by the concave mirror is smaller than the object.
 ③ The image by a concave mirror is always upright.
 ④ The image by a convex mirror is always smaller than the object.
 ⑤ Reflection by a spherical mirror does not follow the laws of reflection.

8. [3 points]

What is the magnitude of the momentum of a photon of wavelength 2.21×10^{-12} m? (Note that Planck's constant is $h = 6.63 \times 10^{-34}$ J · s)

- ① 1.0×10^{-22} kg · m/s ② 1.5×10^{-22} kg · m/s
 ③ 3.0×10^{-22} kg · m/s ④ 1.0×10^{-21} kg · m/s
 ⑤ 1.5×10^{-21} kg · m/s

9. [3 points]

The energy level of the hydrogen atom is

$$E_n = -\frac{13.6}{n^2} \text{ (eV)}, \text{ where } n \text{ is the principal quantum}$$

number. When an electron transition occurs from a state with n_1 to a state with n_2 in a hydrogen atom, in which of the following cases the wavelength of emitted light is the shortest?

- ① $n_1 = 2, n_2 = 1$ ② $n_1 = 3, n_2 = 1$
 ③ $n_1 = 3, n_2 = 2$ ④ $n_1 = 4, n_2 = 3$
 ⑤ $n_1 = 4, n_2 = 2$

Physics Examination(A TYPE) Answers

<Multiple choice Types> There is only one correct answer per each question. Mark your answer choice on the OMR answer sheet.

- For each correct answer, you will get the points indicated next to each question number.
- No penalty point is applied to an incorrect answer.

Answers:

1. ③
2. ④
3. ④
4. ②
5. ⑤
6. ①
7. ④
8. ③
9. ②

Math Examination(TYPE A)

< Multiple choice Types > There is only one correct answer for each question. Mark your choice on the OMR answer sheet.

- The points for each question are listed next to the question number.
- You can use the right side of each page for your memo.

1. [2 points]

Simplify $2^{\log_8 \sqrt{3} \times \log_9 64}$.

- ① 1 ② $\sqrt{2}$ ③ 2 ④ $2\sqrt{2}$ ⑤ 4

2. [2 points]

Simplify $\left(\frac{\sqrt{3}+i}{\sqrt{2}+\sqrt{2}i}\right)^{42}$.

- ① $-i$ ② -1 ③ 1 ④ i ⑤ $1+i$

3. [2 points]

Evaluate $\lim_{x \rightarrow 0} \frac{x \sin x}{1 - \cos x}$.

- ① $\frac{1}{4}$ ② $\frac{1}{2}$ ③ 1 ④ 2 ⑤ 4

4. [2 points]

When $f(x) = x^x$, find $f'(e)$.

- ① 1 ② e^e ③ $2e^e$ ④ $3e^e$ ⑤ $4e^e$

5. [2 points]

Simplify $(\sqrt{2}+1)^4 + (\sqrt{2}-1)^4$.

- ① 31 ② 34 ③ 37 ④ 40 ⑤ 43

6. [3 points]

When $\sin \theta = \frac{1}{3}$, find $|\cos 3\theta|$.

- ① $\frac{4\sqrt{2}}{27}$ ② $\frac{7\sqrt{2}}{27}$ ③ $\frac{10\sqrt{2}}{27}$
 ④ $\frac{13\sqrt{2}}{27}$ ⑤ $\frac{16\sqrt{2}}{27}$

7. [3 points]

When $f(x) = \sin^2 x + 3\cos x$, find

$$\lim_{h \rightarrow 0} \frac{f\left(\frac{\pi}{2}+h\right) - f\left(\frac{\pi}{2}\right)}{h}$$

- ① -3 ② -1 ③ 0 ④ 1 ⑤ 3

8. [3 points]

Evaluate $\sum_{n=1}^{10} \frac{1}{n^3 + 3n^2 + 2n}$.

- ① $\frac{61}{264}$ ② $\frac{21}{88}$ ③ $\frac{65}{264}$
④ $\frac{67}{264}$ ⑤ $\frac{23}{88}$

9. [3 points]

Compute $\int_0^\pi x \cos x \, dx$.

- ① -1 ② $-\frac{\pi}{2}$ ③ -2 ④ $-\pi$ ⑤ -3

10. [3 points]

For a triangle $\triangle ABC$ with $\overline{AB}=7$, $\overline{AC}=8$, $\overline{BC}=13$, find the area of $\triangle ABC$.

- ① $6\sqrt{3}$ ② $8\sqrt{3}$ ③ $10\sqrt{3}$
④ $12\sqrt{3}$ ⑤ $14\sqrt{3}$

11. [4 points]

When $\sum_{k=1}^n a_k = \log_2(n^2 + 2n)$ for $n = 1, 2, 3, \dots$,

find a_{10} .

- ① $\log_2 \frac{10}{33}$ ② $\log_2 \frac{20}{33}$ ③ $\log_2 \frac{10}{11}$
④ $\log_2 \frac{40}{33}$ ⑤ $\log_2 \frac{50}{33}$

12. [4 points]

Simplify $\sin^2\left(\frac{\pi}{5} - \frac{\pi}{3}\right) + \sin^2\frac{\pi}{5} + \sin^2\left(\frac{\pi}{5} + \frac{\pi}{3}\right)$.

- ① $\frac{3}{4}$ ② 1 ③ $\frac{5}{4}$ ④ $\frac{3}{2}$ ⑤ $\frac{7}{4}$

13. [4 points]

When a sequence $\{a_n\}_{n=1}^\infty$ satisfies $a_1 = 2$ and

$$a_{n+1} = 3a_n + 2, \quad n = 1, 2, 3, \dots,$$

find a_{20} .

- ① $3^{20} - 2$ ② $3^{20} - 1$ ③ 3^{20}
④ $3^{20} + 1$ ⑤ $3^{20} + 2$

14. [4 points]

When $f'(x) = (\sqrt{x} + 1)f(x)$ and $f(0) = 1$,

find $f(1)$.

- ① e ② $e^{\frac{4}{3}}$ ③ $e^{\frac{5}{3}}$ ④ e^2 ⑤ $e^{\frac{7}{3}}$

15. [4 points]

When $A^n = \begin{pmatrix} a_n & b_n \\ c_n & d_n \end{pmatrix}$ for $A = \begin{pmatrix} 4 & -2 \\ 3 & -1 \end{pmatrix}$,

find $\sum_{n=1}^{10} (a_n + d_n)$.

- ① $2^{11} + 8$ ② $2^{11} + 10$ ③ $2^{11} + 12$
④ $2^{11} + 14$ ⑤ $2^{11} + 16$

16. [5 points]

Find the distance between the line $3x + 4y - 9 = 0$ and the curve $x^2 + y^2 + 2x + 4y + 4 = 0$.

- ① 1 ② $\sqrt{2}$ ③ $\sqrt{3}$ ④ 2 ⑤ 3

17. [5 points]

When P, Q are two points on the circle $x^2 + y^2 = 4$ at which tangential lines pass through $(3, 4)$, find \overline{PQ} .

- ① $\frac{2\sqrt{21}}{5}$ ② $\frac{4\sqrt{21}}{5}$ ③ $\frac{6\sqrt{21}}{5}$
④ $\frac{8\sqrt{21}}{5}$ ⑤ $2\sqrt{21}$

18. [5 points]

When a differentiable function f satisfies $f(0) = 0$ and

$$xf(x) = x^2 \sin x + x^2 + \int_0^x (x-t)f'(t)dt,$$

find $f(\pi)$.

- ① 2π ② $2\pi + 2$ ③ $2\pi + 4$
④ $2\pi + 6$ ⑤ $2\pi + 8$

19. [5 points]

When $x^2 + 4y^2 = 1$, find the maximum of $x^2 + 4xy + 12y^2$.

- ① 1 ② $\sqrt{2} - 1$ ③ $\sqrt{2}$
④ $1 + \sqrt{2}$ ⑤ $2 + \sqrt{2}$

20. [5 points]

Evaluate $\lim_{n \rightarrow \infty} \sqrt[n]{\frac{(2n)!}{n!n^n}}$.

- ① $\frac{4}{e}$ ② $\frac{2}{e}$ ③ 1 ④ $2e$ ⑤ $4e$

**Solution to 2023 IUT SOCIE Admission Test
(scholarship)**

-type A

1	2	3	4	5	6	7
②	④	④	③	②	③	①
8	9	10	11	12	13	14
③	③	⑤	④	④	②	③
15	16	17	18	19	20	
①	⑤	②	②	⑤	①	

2023 IUT Admission Test(SOCIE)

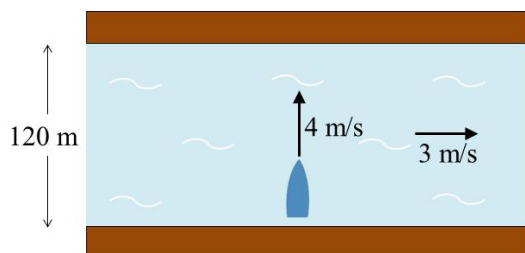
Physics Examination(A TYPE)

<Multiple choice Types> There is only one correct answer per each question. Mark your answer choice on the OMR answer sheet.

- For each correct answer, you will get the points indicated next to each question number.
- No penalty point is applied to an incorrect answer.

1. [1 point]

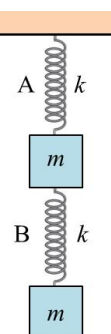
A boat that can move at a speed of 4 m/s in calm water tries to cross a river flowing at a speed of 3 m/s in the shortest time as shown in the figure below. If the river is 120 m wide, how many m does the boat move downstream while reaching the other side of the river?



- ① 60 m
- ② 80 m
- ③ 90 m
- ④ 100 m
- ⑤ 120 m

2. [2 point]

As shown in the figure below, two objects with same mass $m = 1 \text{ kg}$ are suspended from two springs A and B with spring constant $k = 500 \text{ N/m}$. If the stretched lengths of the two springs A and B are x_A and x_B respectively, what is $x_A + x_B$? (Ignore the mass of the springs and assume that the magnitude of the gravitational acceleration is $g = 10 \text{ m/s}^2$.)



- ① 0.02 m
- ② 0.03 m
- ③ 0.04 m
- ④ 0.05 m
- ⑤ 0.06 m

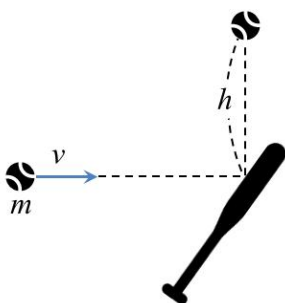
3. [2 point]

What is the minimum work required to move an object of mass m from the earth's surface to a height equal to the Earth's radius R , given that the gravitational acceleration at the earth's surface is g ?

- ① mgR
- ② $\frac{1}{2}mgR$
- ③ $\frac{1}{\sqrt{2}}mgR$
- ④ $\frac{1}{4}mgR$
- ⑤ $2mgR$

4. [2 points]

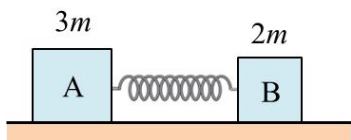
As shown in the figure below, when a ball with mass m flying horizontally at a speed v is hit with a baseball bat, it rises vertically upwards to h and then falls down. What is the magnitude of impulse that the bat exerts on the ball? (The magnitude of the gravitational acceleration is g .)



- ① $m\sqrt{v^2 + 2gh}$ ② $m\sqrt{v^2 + gh}$ ③ $m\sqrt{v^2/2 + gh}$
 ④ $m\sqrt{v^2/2 + 2gh}$ ⑤ $m\sqrt{v^2 + gh/2}$

5. [1 points]

As shown in the figure below, on a frictionless horizontal surface, object A with mass $3m$ and object B with mass $2m$ are in contact with both ends of the spring inbetween, and the spring is compressed. After the spring is released, if the speed of object A is $v_A = 4$ m/s, what is the speed of object B? (Ignore the mass of the spring.)



- ① 3 m/s ② 4 m/s ③ 5 m/s
 ④ 6 m/s ⑤ 8 m/s

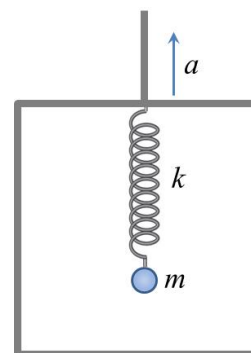
6. [1 point]

For a simple pendulum in which a weight of mass m is suspended from a string of length l , the period of the pendulum is T_0 . If the length of the string is doubled and the mass is tripled, what is the period of this pendulum?

- ① $\sqrt{2} T_0$ ② $\sqrt{3} T_0$ ③ $\sqrt{\frac{2}{3}} T_0$
 ④ $\sqrt{\frac{3}{2}} T_0$ ⑤ $\frac{1}{\sqrt{3}} T_0$

7. [2 points]

As shown in the figure below, an elevator is moving upward with an acceleration $a = 4.0$ m/s². An object of mass $m = 2.0$ kg is suspended in an elevator by a spring with spring constant $k = 200$ N/m. What is the stretched length of the spring? (Ignore the mass of the spring and assume that the magnitude of gravitational acceleration is $g = 10$ m/s²)



- ① 0.08 m ② 0.10 m ③ 0.14 m
 ④ 0.15 m ⑤ 0.18 m

8. [1 point]

30 g of water at 20 °C was mixed with 70 g of water at 60 °C. What is the final temperature of the water?

- ① 45 °C ② 48 °C ③ 50 °C
④ 52 °C ⑤ 54 °C

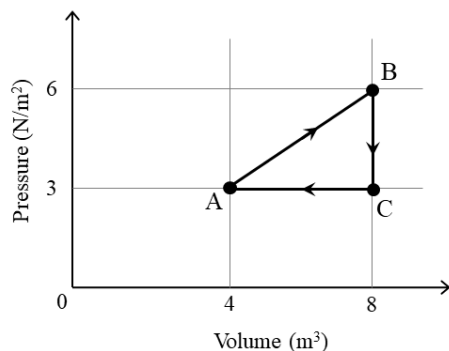
9. [1 points]

An ideal gas has a volume of 3.0 L at 100 K and 1 atm. If the temperature of this gas is raised to 200 K and the pressure is changed to 3.0 atm, what will be its volume?

- ① 1.2 L ② 1.5 L ③ 1.6 L ④ 1.8 L
⑤ 2.0 L

10. [2 points]

As shown in the graph, a certain amount of ideal gas changed its state in the process of $A \rightarrow B \rightarrow C \rightarrow A$. How much heat is absorbed by the gas during this cycle?



- ① 3 J ② 4 J ③ 5 J ④ 6 J ⑤ 8 J

11. [1 point]

How much electrical energy is converted into heat in 1.0 minute by a 20-ohm resistor carrying 0.5 A of current?

- ① 100 J ② 150 J ③ 300 J
④ 450 J ⑤ 600 J

12. [1 point]

How is the capacitance of a parallel-plate capacitor affected when the area of the plates is doubled and the distance between the plates is halved?

- ① 1/4 of its original value
② 1/2 of its original value
③ remains the same
④ 2 times greater
⑤ 4 times greater

13. [1 point]

The energy level diagram for a hypothetical atom is shown as below. The energy for each level is given above ground state. Which of the following photon energies could NOT be emitted from this atom?

$$\begin{aligned} n = 4 & \text{ ————— } E_4 = 7 \text{ eV} \\ n = 3 & \text{ ————— } E_3 = 6 \text{ eV} \\ n = 2 & \text{ ————— } E_2 = 4 \text{ eV} \\ n = 1 & \text{ ————— } E_1 = 0 \end{aligned}$$

- ① 1 eV ② 2 eV ③ 3 eV ④ 4 eV ⑤ 5 eV

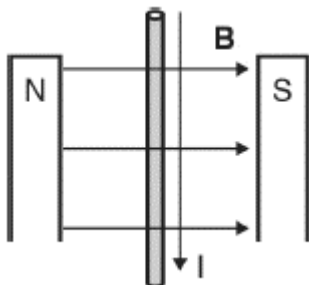
14. [1 point]

In a material, the frequency and the wavelength of light is 4.0×10^{14} Hz and 500 nm, respectively. What is the refractive index of the material? Note that the speed of light in vacuum is 3.0×10^8 m/s.

- ① 1.5 ② 2.0 ③ 2.5 ④ 3.0 ⑤ 4.0

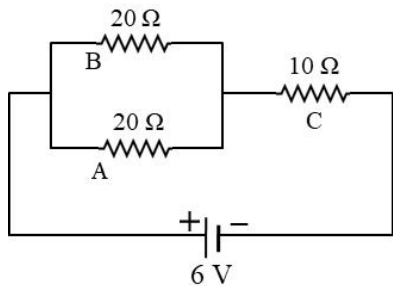
15. [1 point]

A wire carrying a current of 2 A is placed in a magnetic field of 0.1 T as shown below. The length of the wire in the magnetic field is 0.3 m. Determine the magnitude of the force on the wire.



- ① 0.015 N ② 0.06 N ③ 0.67 N ④ 1.5 N
⑤ 6.0 N

16. [2 points] The three resistors A, B, and C are connected in a circuit as shown below. The resistances of the resistors A, B, and C are 20, 20, and 10 Ω , respectively. If the power dissipated by resistor A is P , how much power is dissipated by resistor C?



- ① $0.25P$ ② $0.5P$ ③ P ④ $2P$ ⑤ $4P$

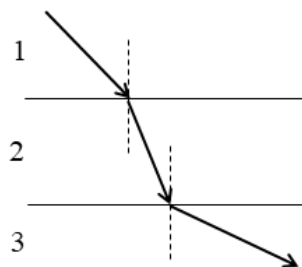
17. [2 points] The figure below shows a transverse wave traveling in the x -axis at a particular instant of time. What is the speed of the wave if the period of the wave is 0.2 s?



- ① 4 cm/s ② 20 cm/s ③ 25 cm/s
④ 50 cm/s ⑤ 100 cm/s

18. [2 points]

In the diagram below, a ray of light refracts as it travels through three media: 1, 2, and 3. The index of refraction for each medium is n_1 , n_2 , and n_3 , respectively. Which of the following correctly ranks the index of refraction from greatest to least?



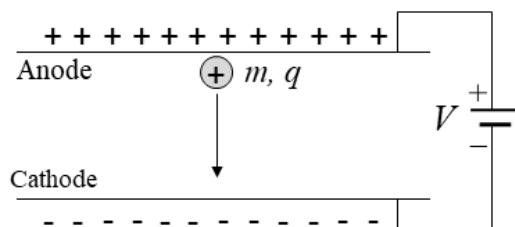
- ① $n_1 > n_2 > n_3$ ② $n_2 > n_1 > n_3$
③ $n_3 > n_1 > n_2$ ④ $n_1 > n_3 > n_2$
⑤ $n_2 > n_3 > n_1$

19. [2 points] What is the kinetic energy of the electron emitted by the photoelectric effect when the wavelength of a light is 820 nm and the work function of a metal is 0.8 eV? (Note that Plank constant h is 4.1×10^{-15} eV·s.)

- ① 0.5 eV ② 0.6 eV ③ 0.7 eV
 ④ 0.8 eV ⑤ 0.9 eV

20. [2 points]

Two parallel metal plates are connected with a battery of voltage V as shown in the figure below. A charged particle with a charge q and a mass m is initially at rest on the positively charged plate (anode) and moves to the negatively charged plate (cathode). The distance between the anode and the cathode is denoted as d . How long does the particle take to arrive at the cathode if $V = 9$ V, $q = 0.04$ C, $d = 0.3$ m, and $m = 5$ g? Neglect the effect of gravitation.



- ① 0.05 s ② 0.1 s ③ 0.2 s
 ④ 0.3 s ⑤ 0.4 s

Physics Examination(A TYPE)

Answers

1. ③
2. ⑤
3. ②
4. ①
5. ④
6. ①
7. ③
8. ②
9. ⑤
10. ④
11. ③
12. ⑤
13. ⑤
14. ①
15. ②
16. ④
17. ④
18. ②
19. ③
20. ①