

## Math Examination(TYPE A)

< Multiple choice Types > There is only one correct answer for each question. Mark your choice on the OMR answer sheet.

- The points for each question are listed next to the question number.
- You can use the right side of each page for your memo.

1. [4 points]

Simplify  $\frac{\sqrt{a}\sqrt{a}}{\sqrt[3]{a}}$  for  $a > 0$ .

- ①  $a^{\frac{1}{12}}$     ②  $a^{\frac{1}{4}}$     ③  $a^{\frac{5}{12}}$     ④  $a^{\frac{7}{12}}$     ⑤  $a^{\frac{3}{4}}$

2. [4 points]

Solve  $\log_{27}x = \frac{4}{3}$ .

- ① 1    ② 3    ③ 9    ④ 27    ⑤ 81

3. [4 points]

When  $\alpha, \beta$  are solutions of  $x^2 - 2x - 1 = 0$ ,  
find  $\alpha^3 + \beta^3$ .

- ① 8    ② 10    ③ 12    ④ 14    ⑤ 16

4. [4 points]

Simplify  $1 + i + i^2 + \dots + i^{50}$ .

- ①  $-i$     ②  $-1$     ③  $0$     ④  $1$     ⑤  $i$

5. [4 points]

Evaluate  $\sum_{n=1}^{20} (2^{n-1} + 2)$ .

- ①  $2^{20} + 35$     ②  $2^{20} + 39$     ③  $2^{20} + 43$   
④  $2^{20} + 47$     ⑤  $2^{20} + 51$

6. [5 points]

Evaluate  $\lim_{x \rightarrow 1} \frac{x^{10} + 2x - 3}{x - 1}$ .

- ① 0    ② 3    ③ 6    ④ 9    ⑤ 12

7. [5 points]

When  $f(x) = \frac{x+2}{x^2+3}$ , find  $f'(1)$ .

- ①  $-\frac{1}{4}$     ②  $-\frac{1}{8}$     ③  $0$     ④  $\frac{1}{8}$     ⑤  $\frac{1}{4}$

8. [5 points]

For a triangle  $\triangle ABC$  with  $\overline{AB}=3$ ,  $\overline{AC}=4$ ,  
 $\angle A = \frac{\pi}{3}$ , find the area of  $\triangle ABC$ .

- ①  $\sqrt{3}$    ②  $2\sqrt{3}$    ③  $3\sqrt{3}$    ④  $4\sqrt{3}$    ⑤  $5\sqrt{3}$

9. [5 points]

When  $x^2 + y^2 = 2$ ,  $|x| + 2y = 1$ , find  $|x| + y$ .

- ①  $\frac{2}{5}$    ②  $\frac{4}{5}$    ③  $\frac{6}{5}$    ④  $\frac{8}{5}$    ⑤ 2

10. [5 points]

Find the minimum of  $(a+2b)\left(\frac{1}{a} + \frac{2}{b}\right)$   
for  $a > 0$ ,  $b > 0$ .

- ① 1   ② 3   ③ 5   ④ 7   ⑤ 9

11. [5 points]

When an arithmetic sequence  $\{a_n\}_{n=1}^{\infty}$  satisfies  
 $a_1 = 1$ ,  $a_7 = 5$ , find  $a_{10}$ .

- ① 6   ② 7   ③ 8   ④ 9   ⑤ 10

12. [5 points]

Find the distance from the point  $(1,2)$  to the line  
 $2x + y + 1 = 0$ .

- ①  $\sqrt{5}$    ②  $\sqrt{6}$    ③  $\sqrt{7}$   
④  $2\sqrt{2}$    ⑤  $\sqrt{10}$

13. [5 points]

When a line  $y = ax + b$  is tangent to the curve  
 $y = x^3 + 5x - 4$  at  $(1,2)$ , find  $2a + b$ .

- ① 2   ② 4   ③ 6   ④ 8   ⑤ 10

14. [5 points]

Find  $\lim_{h \rightarrow 0} \frac{1}{h} \int_1^{1+h} (2x^5 + x^2 + 1) dx$ .

- ① 1   ② 2   ③ 3   ④ 4   ⑤ 5

15. [5 points]

Compute  $\int_1^2 x(2x^2 + 1)^3 dx$ .

- ① 390   ② 395   ③ 400   ④ 405   ⑤ 410

16. [6 points]

When  $A^{-1}(A+B)B^{-1} = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$  for

$A = \begin{pmatrix} 1 & 2 \\ 2 & 3 \end{pmatrix}$ ,  $B = \begin{pmatrix} 1 & 3 \\ 2 & 5 \end{pmatrix}$ , find  $a+b+c+d$ .

- ① -2    ② -1    ③ 0    ④ 1    ⑤ 2

17. [6 points]

When  $f(x) = x^2 + \int_0^2 f(t)dt$ , find  $f(1)$ .

- ①  $-\frac{5}{3}$     ②  $-\frac{1}{3}$     ③ 0  
④  $\frac{1}{3}$     ⑤  $\frac{5}{3}$

18. [6 points]

Find the sum of all solutions of

$$\sin x + \sqrt{3} \cos x = \sqrt{2}, \quad 0 \leq x \leq 2\pi.$$

- ①  $\frac{\pi}{3}$     ②  $\pi$     ③  $\frac{5\pi}{3}$     ④  $\frac{7\pi}{3}$     ⑤  $3\pi$

19. [6 points]

Find the number of all integers  $k$  such that

$f(x) = x^3 + kx^2 + 3x + 5$  is increasing.

- ① 3    ② 5    ③ 7    ④ 9    ⑤ 11

20. [6 points]

Find the area of the region enclosed by two curves

$$y = x^3 + x^2 + 2x + 5, \quad y = x^3 + 2x^2 - x + 7.$$

- ①  $\frac{1}{6}$     ②  $\frac{1}{5}$     ③  $\frac{1}{4}$     ④  $\frac{1}{3}$     ⑤  $\frac{1}{2}$

## Solution to 2023 IUT SBL Pre-university Admission Test

-type A

1	2	3	4	5	6	7
③	⑤	④	⑤	②	⑤	②
8	9	10	11	12	13	14
③	③	⑤	②	①	⑤	④
15	16	17	18	19	20	
④	②	①	④	③	①	

## Math Examination (A TYPE)

<Multiple choice Types> There is only one correct answer per each question. Mark your answer choice on the OMR answer sheet.

- For each correct answer, you will get the points indicated next to each question number.
- No penalty point is applied to an incorrect answer.

1. [8 points]

When  $a = \frac{\sqrt{2+\sqrt{3}}}{\sqrt{2-\sqrt{3}}}$ , find  $a^3 + \frac{1}{a^3}$ .

- ① 20    ② 28    ③ 36    ④ 44    ⑤ 52

2. [8 points]

When  $\alpha, \beta, \gamma$  are solutions of

$x^3 + 3x^2 - 5x + 7 = 0$ , find  $\frac{1}{\alpha^2\beta^2} + \frac{1}{\beta^2\gamma^2} + \frac{1}{\gamma^2\alpha^2}$ .

- ①  $\frac{11}{49}$     ②  $\frac{13}{49}$     ③  $\frac{15}{49}$     ④  $\frac{17}{49}$     ⑤  $\frac{19}{49}$

3. [8 points]

When  $52^x = 16$  and  $13^y = \frac{1}{8}$ , find  $\frac{4}{x} + \frac{3}{y}$ .

- ① 2    ② 4    ③ 6    ④ 8    ⑤ 10

4. [10 points]

Find the sum of all solutions of

$$2\cos^3\theta - 8\cos\theta - 6 = -3\sin^2\theta, \quad 0 \leq \theta \leq 2\pi.$$

- ①  $2\pi$     ②  $\frac{5\pi}{2}$     ③  $3\pi$     ④  $\frac{7\pi}{2}$     ⑤  $4\pi$

5. [10 points]

When  $\sin\alpha = \frac{1}{3}$  and  $\sin\beta = \frac{2}{3}$  for  $0 < \alpha < \frac{\pi}{2}$

and  $\frac{\pi}{2} < \beta < \pi$ , find  $\sin(\alpha + \beta)$ .

- ①  $\frac{\sqrt{2}-\sqrt{5}}{9}$     ②  $\frac{2\sqrt{2}-\sqrt{5}}{9}$   
 ③  $\frac{3\sqrt{2}-\sqrt{5}}{9}$     ④  $\frac{4\sqrt{2}-\sqrt{5}}{9}$   
 ⑤  $\frac{5\sqrt{2}-\sqrt{5}}{9}$

6. [10 points]

When  $f(x) = \frac{x}{\sqrt[3]{x^2+4}}$ , find  $f'(2)$ .

- ①  $\frac{1}{2}$     ②  $\frac{1}{3}$     ③  $\frac{1}{5}$     ④  $\frac{1}{6}$     ⑤  $\frac{1}{8}$

7. [10 points]

When  $y = ax + b$  is the tangent line to  $f(x) = 2x \ln x$  at  $x = e$ , find  $a + b$ .

- ①  $-2e$       ②  $-2e + 2$       ③  $-2e + 4$   
④  $-2e + 6$       ⑤  $-2e + 8$

8. [12 points]

Find  $\int_0^{\ln 3} \frac{e^x}{\sqrt{e^x + 1}} dx$ .

- ①  $4 - 2\sqrt{2}$       ②  $4 - \sqrt{2}$       ③  $4$   
④  $4 + \sqrt{2}$       ⑤  $4 + 2\sqrt{2}$

9. [12 points]

Find the area of the region enclosed by  $y = 2x^3 + x^2 - 4x + 1$  and  $y = 2x^3 - x - 1$ .

- ①  $\frac{1}{2}$       ②  $\frac{1}{3}$       ③  $\frac{1}{4}$       ④  $\frac{1}{6}$       ⑤  $\frac{1}{8}$

10. [12 points]

Find the volume of the solid obtained by revolving the region enclosed by  $y = -x^2 + 1$  and  $y = 0$  about the  $x$ -axis.

- ①  $\frac{11}{15}\pi$       ②  $\frac{16}{15}\pi$       ③  $\frac{7}{5}\pi$   
④  $\frac{26}{15}\pi$       ⑤  $\frac{31}{15}\pi$

## 2023 IUT Pre-University Answer Sheets

[TypeA]

No.	1	2	3	4	5	6	7	8	9	10
Ans.	⑤	⑤	①	③	④	②	③	①	④	②

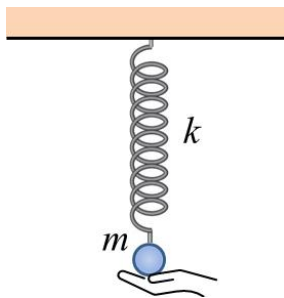
# Physics Examination(A TYPE)

<Multiple choice Types> There is only one correct answer per each question. Mark your answer choice on the OMR answer sheet.

- For each correct answer, you will get the points indicated next to each question number.
- No penalty point is applied to an incorrect answer.

1. [10 points]

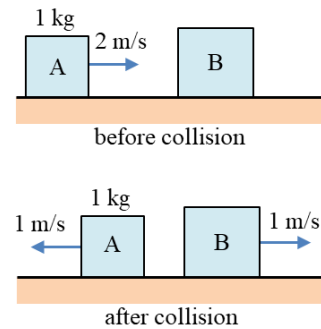
As shown in the figure below, an object of mass  $m$  is attached to the lower end of a spring with spring constant  $k$ , and the object is released when the spring is not stretched at all. How long does it take for the object to reach the lowest position? (Ignore the mass of spring.)



- ①  $2\pi\sqrt{\frac{m}{k}}$
- ②  $\pi\sqrt{\frac{m}{k}}$
- ③  $2\pi\sqrt{\frac{k}{m}}$
- ④  $\pi\sqrt{\frac{k}{m}}$
- ⑤  $\frac{\pi}{2}\sqrt{\frac{k}{m}}$

2. [12 points]

As shown in the figure below, an object A with a mass of 1 kg on a frictionless horizontal surface collides head-on with a stationary object B. Initially the velocity of the object A is 2 m/s to the right. After the collision, the velocities of objects A and B are respectively 1 m/s to the left and 1 m/s to the right. What is the mass of object B?

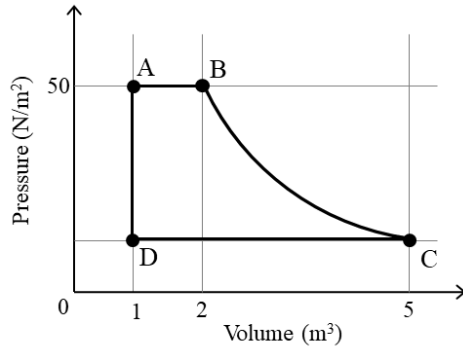


- ① 1 kg
- ② 2 kg
- ③ 3 kg
- ④ 4 kg
- ⑤ 5 kg



3. [12 points]

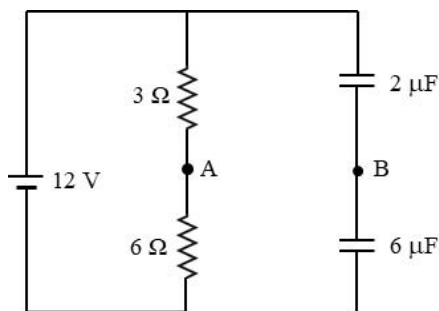
A certain amount of ideal gas changes with a pressure–volume relationship as shown in the graph below. In this graph, pressure is inversely proportional to volume in the B→C process. If the temperature at state B is 300 K, what are the pressure and temperature at state D?



- ① 20 N/m<sup>2</sup>, 60 K      ② 20 N/m<sup>2</sup>, 80 K
- ③ 10 N/m<sup>2</sup>, 60 K      ④ 10 N/m<sup>2</sup>, 80 K
- ⑤ 10 N/m<sup>2</sup>, 100 K

4. [12 points]

As shown in the figure below, two resistors and two capacitors are connected to a 12 V battery. What is the potential difference between point A and point B,  $V_A - V_B$ ?



- ① -3 V      ② 0 V      ③ 3 V
- ④ 4 V      ⑤ 5 V

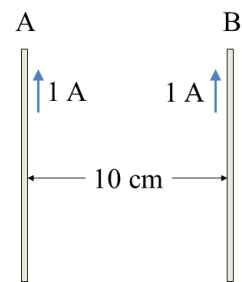
5. [10 points]

If it takes  $3.6 \times 10^{-4}$  J of work to move a charge of  $3.0 \times 10^{-6}$  C from one point to another in the electric field, what is the potential difference between the two points in V?

- ① 12 V      ② 60 V      ③ 120 V
- ④ 150 V      ⑤ 200 V

6. [8 points]

As shown in the figure below, when a current of 1 A flows upward through two parallel wires 10 cm apart, how far is the point where the magnetic field is 0 from wire A?



- ① 2 cm      ② 3 cm      ③ 4 cm
- ④ 5 cm      ⑤ 6 cm

7. [8 points]

Which of the following electromagnetic waves has the shortest wavelength?

- ① visible light      ② x-ray      ③ microwave
- ④ ultraviolet      ⑤ infrared

8. [10 points]

An image of an object placed 60 cm in front of a spherical mirror is formed at a position 180 cm in front of the mirror. What is the focal length of this mirror?

- ① 24 cm      ② 30 cm      ③ 35 cm  
④ 40 cm      ⑤ 45 cm

9. [8 points]

What is the energy of a photon of frequency  $f$ ?  
(Here, Planck constant is  $h$ , and the speed of light in vacuum is  $c$ )

- ①  $hf$       ②  $fc$       ③  $hfc$       ④  $\frac{fc}{h}$       ⑤  $\frac{hf}{c}$

10. [10 points]

The energy level of the ground state of a hydrogen atom is  $-13.6$  eV. What is the energy level of the first excited state?

- ①  $-1.5$  eV      ②  $-2.7$  eV      ③  $-3.4$  eV  
④  $-6.8$  eV      ⑤  $-4.5$  eV

# Physics Examination(A TYPE)

---

**Answers**

1. ②
2. ③
3. ①
4. ⑤
5. ③
6. ④
7. ②
8. ⑤
9. ①
10. ③