# 2024 IUT 1st Admission Test(SBL) Math Examination(TYPE A)

< Multiple choice Types > There is only one correct answer for each question. Mark your choice on the OMR answer sheet. The points for each question are listed next to the question number.

1. [3 points]

Compute  $\frac{3}{\sqrt{11-\sqrt{112}}}$ . (1)  $\sqrt{7}$  (2)  $1+\sqrt{7}$  (3)  $2+\sqrt{7}$ (4)  $3+\sqrt{7}$  (5)  $4+\sqrt{7}$ 

- 2. [3 points] When  $\alpha^2 - \frac{1}{\alpha^2} = 2$  for a real number  $\alpha$ , find  $\alpha^2 + \frac{1}{\alpha^2}$ . (1)  $\frac{\sqrt{2}}{4}$  (2)  $\frac{\sqrt{2}}{2}$  (3)  $\sqrt{2}$  (4)  $2\sqrt{2}$  (5)  $4\sqrt{2}$
- 3. [3 points]

Compute  $\sum_{n=1}^{8} \frac{1}{n^2 + 2n}$ . (1)  $\frac{17}{45}$  (2)  $\frac{19}{45}$  (3)  $\frac{23}{45}$  (4)  $\frac{26}{45}$  (5)  $\frac{29}{45}$ 

4. [3 points] When  $\alpha$ ,  $\beta$ ,  $\gamma$  are the solutions of  $x^3 + 6x^2 + 8x - 2 = 0$ , find  $\frac{\gamma}{\alpha\beta} + \frac{\alpha}{\beta\gamma} + \frac{\beta}{\gamma\alpha}$ . (1) 2 (2) 4 (3) 6 (4) 8 (5) 10 5. [3 points]

Compute  $\log_3 4 \times \log_{\sqrt{5}} 3 \times \log_{\sqrt{2}} 25$ .

- 6. [3 points] When  $\alpha$ ,  $\beta$  satisfy  $2^{\alpha} \cdot 2^{\beta} = 8$  and  $\log_2 \alpha + \log_2 \beta = -2$ , find  $\alpha^2 + \beta^2$ .
  - $(1) \ \frac{11}{2} \ (2) \ \frac{13}{2} \ (3) \ \frac{15}{2} \ (4) \ \frac{17}{2} \ (5) \ \frac{19}{2}$
- 7. [3 points] When  $A = \begin{pmatrix} 2 & -1 \\ 3 & 5 \end{pmatrix}$ ,  $B = \begin{pmatrix} 3 & 1 \\ 2 & 1 \end{pmatrix}$  and  $AB^{-1}A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ , find a + b + c + d. (1) 51 (2) 53 (3) 55 (4) 57 (5) 59
- 8. [3 points] When  $A = \begin{pmatrix} 1 & 0 \\ 3 & -1 \end{pmatrix}, B = \begin{pmatrix} 2 & -3 \\ -1 & 1 \end{pmatrix}$  and  $A (A + B)A^{-1} = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ , find a + b + c + d. (1) -10 (2) -12 (3) -14 (4) -16 (5) -18
- 9. [3 points] When α, β are the solutions of 16<sup>x</sup>-16 · 4<sup>x</sup>+48=0, find α+β.
  ① 2+log<sub>4</sub>3 ② 3+log<sub>4</sub>3 ③ 6+log<sub>4</sub>3
  ④ 8+log<sub>4</sub>3 ⑤ 9+log<sub>4</sub>3

10. [3 points]

When  $f(x) = 4^x + 4^{-x} - 5(2^x + 2^{-x}) + 8$  attains the minimum value b at x = a with  $a \ge 0$ , find a+b.

 $(1) \ \frac{3}{2} \qquad (2) \ \frac{3}{4} \qquad (3) \ \frac{1}{2} \qquad (4) \ \frac{3}{8} \qquad (5) \ \frac{3}{10}$ 

⑤ 20

11. [3 points] When  $\alpha$ ,  $\beta$  are the solutions of  $\log_2 x + 8\log_x 2 = 6$ , find  $\alpha + \beta$ . ① 4 ② 8 ③ 12 ④ 16

12. [3 points]
Find the minimum value of f(x) = 3 | x + 2 | + 4 | x - 1 | .
① 1 ② 3 ③ 5 ④ 7 ⑤ 9

13. [3 points]

Compute  $\left(\frac{\sqrt{2} - \sqrt{2}i}{\sqrt{3} + i}\right)^{12}$ . (1) -1 (2) 1 (3) i (4) 1 - i (5) 1 + i

14. [3 points] When  $x = \frac{\sqrt{3} - i}{\sqrt{3} + i}$  and  $y = \frac{\sqrt{3} + i}{\sqrt{3} - i}$ , find  $\frac{y+1}{x} - \frac{x+1}{y}$ . (1)  $2\sqrt{3}i$  (2)  $4\sqrt{3}i$  (3)  $6\sqrt{3}i$ (4)  $8\sqrt{3}i$  (5)  $10\sqrt{3}i$ 

15. [3 points] When  $\sin x - \cos x = \frac{1}{\sqrt{2}}$ , find  $\sin^3 x - \cos^3 x$ . (1)  $\frac{\sqrt{2}}{8}$  (2)  $\frac{3\sqrt{2}}{8}$  (3)  $\frac{5\sqrt{2}}{8}$ (4)  $\frac{7\sqrt{2}}{8}$  (5)  $\frac{9\sqrt{2}}{8}$  16. [3 points]

Find the sum of all solutions of

$$\cos^{2} \frac{x}{2} + \sin^{2} x = \frac{3}{2} \text{ for } 0 \le x \le \pi.$$

$$(1) \frac{\pi}{2} \quad (2) \frac{2\pi}{3} \quad (3) \frac{3\pi}{4} \quad (4) \frac{5\pi}{6} \quad (5) \pi$$

17. [3 points] Find the sum of all solutions of  $\cos 2x = 2 + 3\sin x$  for  $0 \le x \le 2\pi$ . (1)  $\frac{5\pi}{2}$  (2)  $3\pi$  (3)  $\frac{7\pi}{2}$  (4)  $4\pi$  (5)  $\frac{9\pi}{2}$ 

18. [3 points] Find the number of pairs (x,y) satisfying  $\frac{1}{x} + \frac{1}{y} = \frac{1}{3}$ , where x, y are positive integers. (1) 1 (2) 3 (3) 5 (4) 7 (5) 9

19. [3 points]  
Compute 
$$\lim_{x \to \infty} \frac{\sqrt{4x^2 + 1} - 2x}{\sqrt{x^2 + 1} - x}$$
.  
(1)  $\frac{1}{2}$  (2)  $\frac{1}{3}$  (3)  $\frac{1}{4}$  (4)  $\frac{1}{6}$  (5)  $\frac{1}{8}$ 

20. [3 points] When y = ax + b is the tangent line to  $f(x) = x^5 - 2x^3 - 3x - 1$  at x = 1, find  $a^2 + b^2$ . (1) 11 (2) 13 (3) 15 (4) 17 (5) 19 21. [4 points] When  $\lim_{x \to 1} \frac{f(x)}{x-1} = 9$  for a continuous function f(x), find  $\lim_{x \to 1} \frac{\sqrt{x-1+f(x)}}{x^3-1}$ . (1)  $\frac{11}{6}$  (2)  $\frac{13}{6}$  (3)  $\frac{5}{2}$  (4)  $\frac{17}{6}$  (5)  $\frac{19}{6}$ 

#### 22. <sup>[4</sup> points]

 $\begin{array}{ll} \mbox{When} & x^3+6x^2+9x+k=0\,,\ k\neq 0 & \mbox{has exactly} \\ \mbox{two distinct real solutions } \alpha & \mbox{and } \beta\,,\ \mbox{find } \alpha+\beta\,. \\ \begin{tabular}{ll} \end{tabular} & \end{tabular} \\ \end{tabular} \end{tabular} & \end{tabular} \end{tabul$ 

23. [4 points]

When M and m are the maximum and minimum values of  $f(x) = x^4 - 2x^3 + x^2 + 1$  for  $0 \le x \le 1$ , find M+m.

#### 24. <sup>[4 points]</sup>

Find the farthest distance from (1,0) to a point on  $4x^2 + y^2 = 4$ .

$$(1) \frac{2}{\sqrt{3}} \quad (2) \frac{4}{\sqrt{3}} \quad (3) \frac{6}{\sqrt{3}} \quad (4) \frac{8}{\sqrt{3}} \quad (5) \frac{10}{\sqrt{3}}$$

## 25. [4 points]

When 
$$f(2) = 3$$
,  $f'(2) = 7$  and  
 $g(x) = \frac{\sqrt{f(x)+1}}{3x+2}$ , find  $g'(2)$ .  
 $11\frac{1}{2}$   $21\frac{1}{4}$   $31\frac{1}{8}$   $41\frac{1}{12}$   $51\frac{1}{16}$ 

26. [4 points]  
Compute 
$$\int_0^1 x (2x^2 + 1)^3 dx$$
.  
(1) 5 (2) 7 (3) 9 (4) 12 (5) 14

27. [4 points]

When  $f(x) = \frac{ax^2 + 2x + b}{x^2 + 1}$  has a local maximum f(1) = 5 at x = 1, find  $a^2 + b^2$ . (1) 20 (2) 24 (3) 28 (4) 32 (5) 36

Find the area of the region enclosed by  $y = x^3 + 3x^2 - 2x - 3$  and  $y = x^3 + 2x^2 + 2x - 6$ .

29. [4 points] Compute  $\int_{0}^{2} |x^{2} - 1| dx$ .

 $(1) 2 \qquad (2) 4 \qquad (3) 6 \qquad (4) 8 \qquad (5) 10$ 

30. [4 points]

When a differentiable function f(x) satisfies

$$\int_0^x tf(t) dt = \sqrt{3x^2 + 1} - 1 \text{ for } x > 0, \text{ find } f'(1).$$

$$(1) - \frac{1}{8} \quad (2) - \frac{3}{8} \quad (3) - \frac{5}{8} \quad (4) - \frac{7}{8} \quad (5) - \frac{9}{8}$$

[TypeA]

No.	1	2	3	4	5	6	7	8	9	10
Ans.	3	4	5	5	3	4	2	3	1	2
No.	11	12	13	14	15	16	17	18	19	20
Ans.	5	5	1	1	3	4	5	2	1	4
No.	21	22	23	24	25	26	27	28	29	30
Ans.	5	3	2	2	3	1	4	2	1	5

## 2024 IUT Admission Test(SOCIE, Type A) Math & Physics Examination



8. [3 points]  
When 
$$\begin{pmatrix} 2 & 1 \\ -3 & -1 \end{pmatrix}^{10} = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$$
, find  $a+b+c+d$ .  
(1) -3 (2) -1 (3) (4) (5) (3)

- 9. [3 points]
  - Find the sum of all solutions of  $\cos 2x + 3\sin x + 1 = 0$ for  $0 \le x \le 2\pi$ .

$$(1) \pi \quad (2) \ \frac{3\pi}{2} \qquad (3) \ 2\pi \qquad (4) \ \frac{5\pi}{2} \qquad (5) \ 3\pi$$

- 10. [3 points] When  $a_1 = 2$  and  $a_{n+1} = 3a_n - 2$  for  $n = 1, 2, 3, \dots$ , find  $a_{100}$ .
- 11. [3 points] Find  $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n^2 + n} (\sqrt{n} + \sqrt{n+1})}$ . (1)  $\frac{1}{2}$  (2) 1 (3)  $\frac{3}{2}$  (4) 2 (5)  $\frac{5}{2}$

12. [3 points] Compute  $\int_0^{\pi} \sin x \cos 2x \, dx.$   $(1) -\frac{2}{3} \qquad (2) -\frac{1}{3} \qquad (3) 0$ 

(4) 
$$\frac{1}{3}$$
 (5)  $\frac{2}{3}$ 

13. [4 points] Find the minimum value of  $f(x) = \log_2 x + \log_x 4$ for x > 1.

$$(1) \ \frac{\sqrt{2}}{2} \qquad (2) \ 1 \quad (3) \ \sqrt{2} \qquad (4) \ 2 \quad (5) \ 2\sqrt{2}$$

- 14. [4 points] Compute  $\lim_{n \to \infty} \sum_{k=1}^{n} \frac{1}{n} \ln \left( 1 + \frac{2k}{n} \right)$ . (1)  $\frac{\ln 3 - 2}{8}$  (2)  $\frac{\ln 3 - 2}{4}$  (3)  $\frac{3\ln 3 - 2}{8}$ (4)  $\frac{3\ln 3 - 2}{2}$  (5)  $\frac{5\ln 3 - 2}{8}$
- 15. [4 points] When M,m are the maximum and minimum values of  $f(x) = \frac{x}{x^2 + x + 1}$ , respectively, find M+m.

16. <sup>[4</sup> points]

When the angle between two lines x+2y+1=0, ax+y-2=0is  $\frac{\pi}{4}$ , find the constant *a*, where a > 0. (1) 1 (2) 2 (3) 3 (4) 4 (5) 5

17. [4 points]

Compute 
$$\lim_{h \to 0} \frac{1}{h} \int_{2-h}^{2+h} e^{-x^2} dx.$$
  
(1)  $-e^{-4}$  (2)  $-e^{-4}$  (3) 0  
(4)  $e^{-4}$  (5)  $2e^{-4}$ 

18. [5 points]  
When 
$$f(x) = \begin{cases} ae^x + 2\cos x, & x \le 0\\ b\sin x + 1, & x > 0 \end{cases}$$
 is  
differentiable at  $x = 0$  for constants  $a, b$ , find  
 $a^2 + b^2$ .

(4) 8

⑤ 10

③ 6

② 4

19. [5 points]

① 2

Find the area of the region enclosed by  $y=2x^2-5x+3, \ y=-x^2+7x-6 \ .$ 

$$(1) 3 (2) \frac{7}{2} (3) 4 (4) \frac{9}{2} (5) 5$$

20. [5 points]

When the line y = 2x + 1 meets the curve  $y = ke^x$ at only one point, find the constant k.

When 
$$a_n = \int_0^1 (1-x^2)^n dx$$
, find  $\frac{a_{101}}{a_{100}}$ .  
(1)  $\frac{200}{201}$  (2)  $\frac{201}{202}$  (3)  $\frac{202}{203}$   
(4)  $\frac{203}{204}$  (5)  $\frac{204}{205}$ 

#### 22. [3 points]

When a force acts on a block as shown below, which one has the greatest acceleration? (In the picture, mrepresents mass and F represents force.)



#### 23. [4 points]

A force whose magnitude changes with time is applied for 4 seconds to an object with a mass of 5 kg at rest on a frictionless horizontal surface, as shown in the figure below. What is the speed of the object after 4 seconds?



#### 24. <sup>[3 points]</sup>

When a particle with a mass of 1.0 g and a charge of  $1.0 \times 10^{-5} \text{ C}$  that is at rest at point A accelerates to point B, where the potential is 2 V lower than that of point A, what is the speed of the particle at the moment it reaches point B?

① 0.1 m/s	② 0.2 m/s	③ 0.4 m/s
④ 0.5 m/s	⑤ 1.0 m/s	

#### 25. [3 points]

As shown in the figure below, two infinitely long straight wires A and B, carrying currents of 1 A and 3 A respectively, flowing in the same direction, are 20 cm apart. How far from wire A is the point where the magnetic field strength is 0 between two straight wires?



#### 26. [3 points]

The picture below shows the shape of a wave traveling in the x-axis direction at a certain moment. What is the propagation speed of this wave when its period is 2.0 seconds?



#### 27. [3 points]

When Young's double slit experiment is performed using light with a wavelength of  $4.0 \times 10^{-7}$  m, the gap between the interference patterns is 2.0 mm. If the same experiment is performed with different light and the interference pattern is 3.0 mm apart, what is the wavelength of this light?

- (1)  $1.5 \times 10^{-7}$  m (2)  $3.0 \times 10^{-7}$  m (3)  $4.0 \times 10^{-7}$  m (4)  $5.0 \times 10^{-7}$  m
- (5)  $6.0 \times 10^{-7} \text{ m}$

#### 28. [3 points]

When light with a photon energy of 2W was shined on a metal plate with a work function of W, the maximum speed of the photoelectrons emitted was v. What is the maximum speed of photoelectrons emitted when light with a photon energy of 5W is shined on the same metal plate?

3 3v

#### 29. [3 points]

Two objects A and B, each with a mass of 100 g, are heated equally with a heater of the same power and the temperature is measured at regular time intervals to obtain results as shown in the graph below. What is the ratio  $c_A/c_B$  of the specific heats of the two objects? (However, ignore heat loss that may occur when heating the objects.)



#### 30. [5 points]

A uniform magnetic field B oriented perpendicular to the xy-plane exists in the region where  $x \ge 0$ . The direction of the magnetic field is into the page as shown in the figure below. A particle with mass m and charge q(q > 0) is incident at a speed v at an angle of 45° to the x-axis from the origin. If the coordinates of the point where the particle intersects the y-axis are  $(0, y_1)$ , what is the value of  $y_1$ ?



1	2	3	4	5	6	7
3	1	2	1	4	2	5
8	9	10	11	12	13	14
4	5	4	2	1	5	4
15	16	17	18	19	20	21
1	3	5	1	3	2	3
22	23	24	25	26	27	28
5	3	2	4	3	5	1
29	30					
2	(4)					

### 2024 IUT Admission Test(SOCIE) MATH / Physics