

2025 IUT 2nd Admission Test(SBL Scholarship)

Math Examination(TYPE A)

< Multiple choice Types > There is only one correct answer for each question. Mark your choice on the OMR answer sheet.

- The points for each question are listed next to the question number.
- You can use the right side of each page for your memo.

1. [2 points]

Simplify $\frac{1}{\sqrt{7+2\sqrt{12}}}$.

- ① $2 - \sqrt{3}$ ② $2 - 2\sqrt{3}$ ③ 2
 ④ $2 + \sqrt{3}$ ⑤ $2 + 2\sqrt{3}$

2. [2 points]

Find $\frac{1}{(1-\sqrt{2})^2} - \frac{1}{(1+\sqrt{2})^2}$.

- ① $2\sqrt{2}$ ② $4\sqrt{2}$ ③ $6\sqrt{2}$
 ④ $8\sqrt{2}$ ⑤ $10\sqrt{2}$

3. [2 points]

Simplify $\sqrt{2} \times \sqrt[3]{4} \div \sqrt[4]{8}$.

- ① $\sqrt[4]{2}$ ② $\sqrt[3]{2}$ ③ $\sqrt[12]{2^5}$ ④ $\sqrt{2}$ ⑤ $\sqrt[12]{2^7}$

4. [2 points]

When $x^{\frac{1}{3}} + x^{-\frac{1}{3}} = 3$, find $x^2 + x^{-2}$.

- ① 316 ② 318 ③ 320 ④ 322 ⑤ 324

5. [2 points]

Simplify $\frac{\sqrt{2} - \sqrt{3} - \sqrt{4} + \sqrt{6}}{\sqrt{2} + \sqrt{3} - \sqrt{4} - \sqrt{6}}$.

- ① $2\sqrt{6} - 1$ ② $2\sqrt{6} - 2$ ③ $2\sqrt{6} - 3$
 ④ $2\sqrt{6} - 4$ ⑤ $2\sqrt{6} - 5$

6. [2 points]

Simplify $\frac{1}{(a+1)(a^2+1)(a^4+1)(a^8+1)}$ when $a = \sqrt[8]{3}$.

- ① $\frac{\sqrt[8]{3}-1}{2}$ ② $\frac{\sqrt[8]{3}-1}{4}$ ③ $\frac{\sqrt[8]{3}-1}{6}$
 ④ $\frac{\sqrt[8]{3}-1}{8}$ ⑤ $\frac{\sqrt[8]{3}-1}{10}$

7. [2 points]

When α, β, γ are the solutions of $x^3 - 3x^2 - 5x + 6 = 0$, find

$\frac{\beta+\gamma}{\alpha} + \frac{\gamma+\alpha}{\beta} + \frac{\alpha+\beta}{\gamma}$.

- ① $-\frac{1}{2}$ ② $-\frac{3}{2}$ ③ $-\frac{5}{2}$
 ④ $-\frac{7}{2}$ ⑤ $-\frac{9}{2}$

8. [2 points]

When α and β are solutions of $x^2 - 2x + 6 = 0$,

find $\frac{\beta^2}{\alpha} + \frac{\alpha^2}{\beta}$.

- ① $-\frac{2}{3}$ ② $-\frac{5}{3}$ ③ $-\frac{8}{3}$
 ④ $-\frac{11}{3}$ ⑤ $-\frac{14}{3}$

9. [2 points]

When $4^{2^x} = 27$ and $14^y = \frac{1}{81}$, find $\frac{3}{2x} + \frac{2}{y}$.

- ① $\frac{1}{2}$ ② $\frac{1}{3}$ ③ $\frac{1}{6}$ ④ $\frac{1}{8}$ ⑤ $\frac{1}{9}$

10. [2 points]

Find $\sum_{n=2}^{10} \frac{1}{2^n}$.

- ① $\frac{63}{128}$ ② $\frac{127}{256}$ ③ $\frac{255}{512}$ ④ $\frac{511}{1024}$ ⑤ $\frac{1023}{2048}$

11. [2 points]

Find $\log_2 \frac{3}{2} + \log_2 \frac{4}{3} + \log_2 \frac{5}{4} + \dots + \log_2 \frac{32}{31}$.

- ① 2 ② 3 ③ 4 ④ 5 ⑤ 6

12. [2 points]

Find the maximum value of

$$f(x) = 2\sqrt{2} \sin x + 2\sqrt{2} \cos x + 3 \cos\left(x + \frac{\pi}{4}\right).$$

- ① 4 ② 5 ③ 6 ④ 7 ⑤ 8

13. [2 points]

When $A = \begin{pmatrix} 2 & -1 \\ 1 & -1 \end{pmatrix}$ and $A^2 + A^{-1} = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$,

find $a + b + c + d$.

- ① 2 ② 4 ③ 6 ④ 8 ⑤ 10

14. [2 points]

When $x + y = 3$ and $x^2 + y^2 = 7$, find $x^3 + y^3$.

- ① 6 ② 9 ③ 12 ④ 15 ⑤ 18

15. [2 points]

Find the sum of all real solutions of

$$4^x - 3 \cdot 2^x + \frac{12}{2^x} = 4.$$

- ① $\log_2 3$ ② $\log_2 5$ ③ $\log_2 6$
④ $\log_2 8$ ⑤ $\log_2 10$

16. [2 points]

When $z = \frac{\sqrt{2+\sqrt{2}}}{2} - \frac{i\sqrt{2-\sqrt{2}}}{2}$, find $z^8 + z^4$.

- ① 0 ② 1 ③ i ④ $1+i$ ⑤ $-1-i$

17. [2 points]

When ω is a root of $x^2 + x + 1 = 0$, find

$$\omega^{2026} + \omega^{20}.$$

- ① -1 ② $-i$ ③ $-1-i$
④ $-2-i$ ⑤ $-2-2i$

18. [2 points]

When $\omega = \frac{\sqrt{3}-i}{2}$, find $\frac{\omega^{12}+1}{\omega^3+1}$.

- ① 1 ② -1 ③ i ④ $1+i$ ⑤ $1-i$

19. [2 points]

Find the minimum value of a for a positive constant $a > 0$ such that two graphs

$$y = \frac{x^2+4}{x} \text{ and } y = a \text{ meet.}$$

- ① 3 ② 4 ③ 5 ④ 6 ⑤ 7

20. [2 points]

When $2^x = 3^y = 36^z$, find $\frac{z}{x} + \frac{z}{y}$.

- ① $\frac{1}{2}$ ② $\frac{1}{3}$ ③ $\frac{1}{4}$ ④ $\frac{1}{6}$ ⑤ $\frac{1}{12}$

21. [2 points]

When $\lim_{x \rightarrow 2} \frac{x^2 - 4}{x^2 + ax + b} = \frac{4}{5}$, find $a + b$.

- ① -1 ② -2 ③ -3 ④ -4 ⑤ -5

22. [2 points]

When $\sin \theta = \frac{1}{\sqrt{3}}$ ($\frac{\pi}{2} < \theta < \pi$),

find $\sin 2\theta + \cos 2\theta$.

- ① $\frac{1 - \sqrt{2}}{3}$ ② $\frac{1 - 2\sqrt{2}}{3}$ ③ $\frac{1 - 3\sqrt{2}}{3}$
④ $\frac{1 - 4\sqrt{2}}{3}$ ⑤ $\frac{1 - 5\sqrt{2}}{3}$

23. [2 points]

Find the sum of all solutions to

$3 \cos 2\theta + 10 \cos \theta = 1$ for $0 \leq \theta < 2\pi$.

- ① 0 ② π ③ 2π ④ 3π ⑤ 4π

24. [2 points]

Find $\sin \frac{\pi}{16} \cos \frac{\pi}{16}$.

- ① $\frac{\sqrt{2 - \sqrt{2}}}{3}$ ② $\frac{\sqrt{2 - \sqrt{2}}}{4}$ ③ $\frac{\sqrt{2 - \sqrt{2}}}{5}$
④ $\frac{\sqrt{2 - \sqrt{2}}}{6}$ ⑤ $\frac{\sqrt{2 - \sqrt{2}}}{7}$

25. [2 points]

Find $\lim_{x \rightarrow 0} \frac{x^2 - \sin(2x^2)}{1 - \cos x}$.

- ① $-\frac{2}{3}$ ② $-\frac{1}{2}$ ③ -1
④ $-\frac{3}{2}$ ⑤ -2

26. [3 points]

When $z = \frac{1+i}{1-i}$, find $\frac{z}{1+z+z^2+\dots+z^{2025}}$.

- ① i ② $-i$ ③ $\frac{1}{2}$ ④ $\frac{1-i}{2}$ ⑤ $\frac{1+i}{2}$

27. [3 points]

When $f(x) = \sin\left(\frac{3+2x-3x^2}{3}\right)$ ($0 \leq x \leq 1$) has

the maximum value $M = \sin(a)$ at $x = b$, find $a + b$.

- ① $\frac{10}{9}$ ② $\frac{13}{9}$ ③ $\frac{16}{9}$ ④ $\frac{19}{9}$ ⑤ $\frac{22}{9}$

28. [3 points]

Find $\lim_{x \rightarrow \infty} (\sqrt{2x^2 + 3x + 7} - \sqrt{2x^2 + x - 4})$.

- ① $\frac{1}{\sqrt{3}}$ ② $\frac{2}{\sqrt{3}}$ ③ $\frac{1}{\sqrt{2}}$ ④ 1 ⑤ $\sqrt{2}$

29. [3 points]

When $A = \begin{pmatrix} 1 & 2 \\ 2 & 3 \end{pmatrix}$, $B = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ and

$ABA^{-1} = \begin{pmatrix} 1 & 0 \\ 2 & 1 \end{pmatrix}$, find $a + b + c + d$.

- ① 2 ② 4 ③ 6 ④ 8 ⑤ 10

30. [3 points]

Find the minimum value of

$f(x) = \frac{3}{2}x^4 + 2x^3 + 3x^2 - 18x + 4$.

- ① $-\frac{11}{2}$ ② $-\frac{13}{2}$ ③ $-\frac{15}{2}$
④ $-\frac{17}{2}$ ⑤ $-\frac{19}{2}$

31. [3 points]

When M and m are the maximum and minimum values of $f(x) = \frac{1}{3}x^3 - 4x + 7$, ($-1 \leq x \leq 3$), find $M + m$.

- ① $\frac{31}{3}$ ② 11 ③ $\frac{35}{3}$ ④ $\frac{37}{3}$ ⑤ 13

32. [3 points]

When $y = ax + b$ is the tangent line to $y = \frac{-2x + 7}{x^2 + x + 3}$ at $x = -1$, find $a + b$.

- ① $\frac{11}{3}$ ② $\frac{13}{3}$ ③ 5 ④ $\frac{17}{3}$ ⑤ $\frac{19}{3}$

33. [3 points]

When $f(x) = \frac{\sqrt{2x+5}}{x^2-x+7}$, find $f'(2)$.

- ① $-\frac{2}{27}$ ② $-\frac{4}{27}$ ③ $-\frac{2}{9}$ ④ $-\frac{8}{27}$ ⑤ $-\frac{10}{27}$

34. [3 points]

Find $\int_{-1}^3 (x^2 - 4x - 1) dx$.

- ① -10 ② $-\frac{32}{3}$ ③ $-\frac{34}{3}$
④ -12 ⑤ $-\frac{38}{32}$

35. [3 points]

Find $\int_0^1 (2x-1)^8 dx$.

- ① $\frac{1}{3}$ ② $\frac{1}{4}$ ③ $\frac{1}{6}$ ④ $\frac{1}{8}$ ⑤ $\frac{1}{9}$

36. [4 points]

Find $\int_0^1 \frac{2x^2 + 1}{\sqrt{2x^3 + 3x + 4}} dx$.

- ① 1 ② $\frac{2}{3}$ ③ $\frac{4}{3}$ ④ 2 ⑤ $\frac{8}{3}$

37. [4 points]

When

$c_n = \frac{1}{\sqrt{n^2+n}} + \frac{1}{\sqrt{n^2+2n}} + \dots + \frac{1}{\sqrt{n^2+n^2}}$, find

$\lim_{n \rightarrow \infty} c_n$.

- ① $\sqrt{2} - 1$ ② $\sqrt{2} - 2$ ③ $2\sqrt{2} - 1$
④ $2\sqrt{2} - 2$ ⑤ $3\sqrt{2} - 1$

38. [4 points]

Find the region of the area enclosed by

$x^3 - 3x^2 + 2x - 5 = 0$ and $x^3 - 4x^2 + 3x + 1 = 0$

- ① $\frac{121}{6}$ ② $\frac{41}{2}$ ③ $\frac{125}{6}$ ④ $\frac{127}{6}$ ⑤ $\frac{43}{2}$

39. [4 points]

When $f(x)$ satisfies

$f(x) = \frac{5}{3}x^3 - 6x^2 + 9x - \int_0^x f'(2t-3) dt$,

find $f'(3)$.

- ① 1 ② 3 ③ 5 ④ 7 ⑤ 9

40. [4 points]

When $\int_1^x tf(t) dt = \frac{4}{3}x^3 - 2x^2 + ax + b$ and

$f(2) = 5$ for a continuous function $f(x)$,

find ab .

- ① $-\frac{2}{3}$ ② $-\frac{4}{3}$ ③ -2 ④ $-\frac{8}{3}$ ⑤ $-\frac{10}{3}$

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Math Examination(TYPE B)

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1. [2 points]

Simplify $\frac{\sqrt{2}-\sqrt{3}-\sqrt{4}+\sqrt{6}}{\sqrt{2}+\sqrt{3}-\sqrt{4}-\sqrt{6}}$.

- ① $2\sqrt{6}-1$ ② $2\sqrt{6}-2$ ③ $2\sqrt{6}-3$
 ④ $2\sqrt{6}-4$ ⑤ $2\sqrt{6}-5$

2. [2 points]

Simplify $\frac{1}{\sqrt{7+2\sqrt{12}}}$.

- ① $2-\sqrt{3}$ ② $2-2\sqrt{3}$ ③ 2
 ④ $2+\sqrt{3}$ ⑤ $2+2\sqrt{3}$

3. [2 points]

Find $\frac{1}{(1-\sqrt{2})^2} - \frac{1}{(1+\sqrt{2})^2}$.

- ① $2\sqrt{2}$ ② $4\sqrt{2}$ ③ $6\sqrt{2}$
 ④ $8\sqrt{2}$ ⑤ $10\sqrt{2}$

4. [2 points]

Simplify $\sqrt{2} \times \sqrt[3]{4} \div \sqrt[4]{8}$.

- ① $\frac{4\sqrt{2}}{\sqrt[12]{2^7}}$ ② $\sqrt[3]{2}$ ③ $\sqrt[12]{2^5}$ ④ $\sqrt{2}$ ⑤

5. [2 points]

When $x^{\frac{1}{3}} + x^{-\frac{1}{3}} = 3$, find $x^2 + x^{-2}$.

- ① 316 ② 318 ③ 320 ④ 322 ⑤ 324

6. [2 points]

Find $\sum_{n=2}^{10} \frac{1}{2^n}$.

- ① $\frac{63}{128}$ ② $\frac{127}{256}$ ③ $\frac{255}{512}$ ④ $\frac{511}{1024}$ ⑤ $\frac{1023}{2048}$

7. [2 points]

Simplify $\frac{1}{(a+1)(a^2+1)(a^4+1)(a^8+1)}$ when $a = \sqrt[8]{3}$.

- ① $\frac{\sqrt[8]{3}-1}{2}$ ② $\frac{\sqrt[8]{3}-1}{4}$ ③ $\frac{\sqrt[8]{3}-1}{6}$
 ④ $\frac{\sqrt[8]{3}-1}{8}$ ⑤ $\frac{\sqrt[8]{3}-1}{10}$

8. [2 points]

When α, β, γ are the solutions of $x^3 - 3x^2 - 5x + 6 = 0$, find

$\frac{\beta+\gamma}{\alpha} + \frac{\gamma+\alpha}{\beta} + \frac{\alpha+\beta}{\gamma}$.

- ① $-\frac{1}{2}$ ② $-\frac{3}{2}$ ③ $-\frac{5}{2}$
 ④ $-\frac{7}{2}$ ⑤ $-\frac{9}{2}$

9. [2 points]

When α and β are solutions of $x^2 - 2x + 6 = 0$,

find $\frac{\beta^2}{\alpha} + \frac{\alpha^2}{\beta}$.

- ① $-\frac{2}{3}$ ② $-\frac{5}{3}$ ③ $-\frac{8}{3}$
 ④ $-\frac{11}{3}$ ⑤ $-\frac{14}{3}$

10. [2 points]

When $42^x = 27$ and $14^y = \frac{1}{81}$, find $\frac{3}{2x} + \frac{2}{y}$.

- ① $\frac{1}{2}$ ② $\frac{1}{3}$ ③ $\frac{1}{6}$ ④ $\frac{1}{8}$ ⑤ $\frac{1}{9}$

11. [2 points]

Find the sum of all real solutions of

$$4^x - 3 \cdot 2^x + \frac{12}{2^x} = 4.$$

- ① $\log_2 3$ ② $\log_2 5$ ③ $\log_2 6$
④ $\log_2 8$ ⑤ $\log_2 10$

12. [2 points]

Find $\log_2 \frac{3}{2} + \log_2 \frac{4}{3} + \log_2 \frac{5}{4} + \dots + \log_2 \frac{32}{31}$.

- ① 2 ② 3 ③ 4 ④ 5 ⑤ 6

13. [2 points]

Find the maximum value of

$$f(x) = 2\sqrt{2} \sin x + 2\sqrt{2} \cos x + 3 \cos\left(x + \frac{\pi}{4}\right).$$

- ① 4 ② 5 ③ 6 ④ 7 ⑤ 8

14. [2 points]

When $A = \begin{pmatrix} 2 & -1 \\ 1 & -1 \end{pmatrix}$ and $A^2 + A^{-1} = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$,

find $a + b + c + d$.

- ① 2 ② 4 ③ 6 ④ 8 ⑤ 10

15. [2 points]

When $x + y = 3$ and $x^2 + y^2 = 7$, find $x^3 + y^3$.

- ① 6 ② 9 ③ 12 ④ 15 ⑤ 18

16. [2 points]

When $2^x = 3^y = 36^z$, find $\frac{z}{x} + \frac{z}{y}$.

- ① $\frac{1}{2}$ ② $\frac{1}{3}$ ③ $\frac{1}{4}$ ④ $\frac{1}{6}$ ⑤ $\frac{1}{12}$

17. [2 points]

When $z = \frac{\sqrt{2+\sqrt{2}}}{2} - \frac{i\sqrt{2-\sqrt{2}}}{2}$, find $z^8 + z^4$.

- ① 0 ② 1 ③ i ④ $1+i$ ⑤ $-1-i$

18. [2 points]

When ω is a root of $x^2 + x + 1 = 0$, find

$$\omega^{2026} + \omega^{20}.$$

- ① -1 ② $-i$ ③ $-1-i$
④ $-2-i$ ⑤ $-2-2i$

19. [2 points]

When $\omega = \frac{\sqrt{3}-i}{2}$, find $\frac{\omega^{12}+1}{\omega^3+1}$.

- ① 1 ② -1 ③ i ④ $1+i$ ⑤ $1-i$

20. [2 points]

Find the minimum value of a for a positive constant $a > 0$ such that two graphs

$$y = \frac{x^2+4}{x} \text{ and } y = a \text{ meet.}$$

- ① 3 ② 4 ③ 5 ④ 6 ⑤ 7

21. [2 points]

Find $\lim_{x \rightarrow 0} \frac{x^2 - \sin(2x^2)}{1 - \cos x}$.

- ① $-\frac{2}{3}$ ② $-\frac{1}{2}$ ③ -1
④ $-\frac{3}{2}$ ⑤ -2

22. [2 points]

When $\lim_{x \rightarrow 2} \frac{x^2 - 4}{x^2 + ax + b} = \frac{4}{5}$, find $a + b$.

- ① -1 ② -2 ③ -3 ④ -4 ⑤ -5

23. [2 points]

When $\sin \theta = \frac{1}{\sqrt{3}}$ ($\frac{\pi}{2} < \theta < \pi$),

find $\sin 2\theta + \cos 2\theta$.

- ① $\frac{1 - \sqrt{2}}{3}$ ② $\frac{1 - 2\sqrt{2}}{3}$ ③ $\frac{1 - 3\sqrt{2}}{3}$
④ $\frac{1 - 4\sqrt{2}}{3}$ ⑤ $\frac{1 - 5\sqrt{2}}{3}$

24. [2 points]

Find the sum of all solutions to

$3 \cos 2\theta + 10 \cos \theta = 1$ for $0 \leq \theta < 2\pi$.

- ① 0 ② π ③ 2π ④ 3π ⑤ 4π

25. [2 points]

Find $\sin \frac{\pi}{16} \cos \frac{\pi}{16}$.

- ① $\frac{\sqrt{2 - \sqrt{2}}}{3}$ ② $\frac{\sqrt{2 - \sqrt{2}}}{4}$ ③ $\frac{\sqrt{2 - \sqrt{2}}}{5}$
④ $\frac{\sqrt{2 - \sqrt{2}}}{6}$ ⑤ $\frac{\sqrt{2 - \sqrt{2}}}{7}$

26. [3 points]

Find the minimum value of

$f(x) = \frac{3}{2}x^4 + 2x^3 + 3x^2 - 18x + 4$.

- ① $-\frac{11}{2}$ ② $-\frac{13}{2}$ ③ $-\frac{15}{2}$
④ $-\frac{17}{2}$ ⑤ $-\frac{19}{2}$

27. [3 points]

When $z = \frac{1+i}{1-i}$, find $\frac{z}{1+z+z^2+\dots+z^{2025}}$.

- ① i ② $-i$ ③ $\frac{1}{2}$ ④ $\frac{1-i}{2}$ ⑤ $\frac{1+i}{2}$

28. [3 points]

When $f(x) = \sin\left(\frac{3+2x-3x^2}{3}\right)$ ($0 \leq x \leq 1$) has

the maximum value $M = \sin(a)$ at $x = b$, find $a + b$.

- ① $\frac{10}{9}$ ② $\frac{13}{9}$ ③ $\frac{16}{9}$ ④ $\frac{19}{9}$ ⑤ $\frac{22}{9}$

29. [3 points]

Find $\lim_{x \rightarrow \infty} (\sqrt{2x^2 + 3x + 7} - \sqrt{2x^2 + x - 4})$.

- ① $\frac{1}{\sqrt{3}}$ ② $\frac{2}{\sqrt{3}}$ ③ $\frac{1}{\sqrt{2}}$ ④ 1 ⑤ $\sqrt{2}$

30. [3 points]

When $A = \begin{pmatrix} 1 & 2 \\ 2 & 3 \end{pmatrix}$, $B = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ and

$ABA^{-1} = \begin{pmatrix} 1 & 0 \\ 2 & 1 \end{pmatrix}$, find $a + b + c + d$.

- ① 2 ② 4 ③ 6 ④ 8 ⑤ 10

31. [3 points]

Find $\int_0^1 (2x-1)^8 dx$.

- ① $\frac{1}{3}$ ② $\frac{1}{4}$ ③ $\frac{1}{6}$ ④ $\frac{1}{8}$ ⑤ $\frac{1}{9}$

32. [3 points]

When M and m are the maximum and minimum values of $f(x) = \frac{1}{3}x^3 - 4x + 7$, ($-1 \leq x \leq 3$), find $M + m$.

- ① $\frac{31}{3}$ ② 11 ③ $\frac{35}{3}$ ④ $\frac{37}{3}$ ⑤ 13

33. [3 points]

When $y = ax + b$ is the tangent line to

$$y = \frac{-2x + 7}{x^2 + x + 3} \text{ at } x = -1, \text{ find } a + b.$$

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34. [3 points]

When $f(x) = \frac{\sqrt{2x+5}}{x^2-x+7}$, find $f'(2)$.

- ① $-\frac{2}{27}$ ② $-\frac{4}{27}$ ③ $-\frac{2}{9}$ ④ $-\frac{8}{27}$ ⑤ $-\frac{10}{27}$

35. [3 points]

Find $\int_{-1}^3 (x^2 - 4x - 1) dx$.

- ① -10 ② $-\frac{32}{3}$ ③ $-\frac{34}{3}$
④ -12 ⑤ $-\frac{38}{32}$

36. [4 points]

When $\int_1^x tf(t) dt = \frac{4}{3}x^3 - 2x^2 + ax + b$ and

$f(2) = 5$ for a continuous function $f(x)$, find ab .

- ① $-\frac{2}{3}$ ② $-\frac{4}{3}$ ③ -2 ④ $-\frac{8}{3}$ ⑤ $-\frac{10}{3}$

37. [4 points]

Find $\int_0^1 \frac{2x^2 + 1}{\sqrt{2x^3 + 3x + 4}} dx$.

- ① 1 ② $\frac{2}{3}$ ③ $\frac{4}{3}$ ④ 2 ⑤ $\frac{8}{3}$

38. [4 points]

When

$$c_n = \frac{1}{\sqrt{n^2 + n}} + \frac{1}{\sqrt{n^2 + 2n}} + \dots + \frac{1}{\sqrt{n^2 + n^2}}, \text{ find}$$

$\lim_{n \rightarrow \infty} c_n$.

- ① $\sqrt{2} - 1$ ② $\sqrt{2} - 2$ ③ $2\sqrt{2} - 1$
④ $2\sqrt{2} - 2$ ⑤ $3\sqrt{2} - 1$

39. [4 points]

Find the region of the area enclosed by

$$x^3 - 3x^2 + 2x - 5 = 0 \text{ and } x^3 - 4x^2 + 3x + 1 = 0$$

- ① $\frac{121}{6}$ ② $\frac{41}{2}$ ③ $\frac{125}{6}$ ④ $\frac{127}{6}$ ⑤ $\frac{43}{2}$

40. [4 points]

When $f(x)$ satisfies

$$f(x) = \frac{5}{3}x^3 - 6x^2 + 9x - \int_0^x f'(2t-3) dt,$$

find $f'(3)$.

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Math Examination(TYPE C)

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1. [2 points]

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2. [2 points]

Simplify $\frac{\sqrt{2} - \sqrt{3} - \sqrt{4} + \sqrt{6}}{\sqrt{2} + \sqrt{3} - \sqrt{4} - \sqrt{6}}$.

- ① $2\sqrt{6} - 1$ ② $2\sqrt{6} - 2$ ③ $2\sqrt{6} - 3$
 ④ $2\sqrt{6} - 4$ ⑤ $2\sqrt{6} - 5$

3. [2 points]

Simplify $\frac{1}{\sqrt{7+2\sqrt{12}}}$.

- ① $2 - \sqrt{3}$ ② $2 - 2\sqrt{3}$ ③ 2
 ④ $2 + \sqrt{3}$ ⑤ $2 + 2\sqrt{3}$

4. [2 points]

Find $\frac{1}{(1-\sqrt{2})^2} - \frac{1}{(1+\sqrt{2})^2}$.

- ① $2\sqrt{2}$ ② $4\sqrt{2}$ ③ $6\sqrt{2}$
 ④ $8\sqrt{2}$ ⑤ $10\sqrt{2}$

5. [2 points]

Simplify $\sqrt{2} \times \sqrt[3]{4} \div \sqrt[4]{8}$.

- ① $\sqrt[4]{2}$ ② $\sqrt[3]{2}$ ③ $\sqrt[12]{2^5}$ ④ $\sqrt{2}$ ⑤ $\sqrt[12]{2^7}$

6. [2 points]

When $42^x = 27$ and $14^y = \frac{1}{81}$, find $\frac{3}{2x} + \frac{2}{y}$.

- ① $\frac{1}{2}$ ② $\frac{1}{3}$ ③ $\frac{1}{6}$ ④ $\frac{1}{8}$ ⑤ $\frac{1}{9}$

7. [2 points]

Find $\sum_{n=2}^{10} \frac{1}{2^n}$.

- ① $\frac{63}{128}$ ② $\frac{127}{256}$ ③ $\frac{255}{512}$ ④ $\frac{511}{1024}$ ⑤ $\frac{1023}{2048}$

8. [2 points]

Simplify $\frac{1}{(a+1)(a^2+1)(a^4+1)(a^8+1)}$ when

$a = \sqrt[5]{3}$.

- ① $\frac{\sqrt[5]{3}-1}{2}$ ② $\frac{\sqrt[5]{3}-1}{4}$ ③ $\frac{\sqrt[5]{3}-1}{6}$
 ④ $\frac{\sqrt[5]{3}-1}{8}$ ⑤ $\frac{\sqrt[5]{3}-1}{10}$

9. [2 points]

When α, β, γ are the solutions of $x^3 - 3x^2 - 5x + 6 = 0$, find

$\frac{\beta+\gamma}{\alpha} + \frac{\gamma+\alpha}{\beta} + \frac{\alpha+\beta}{\gamma}$.

- ① $-\frac{1}{2}$ ② $-\frac{3}{2}$ ③ $-\frac{5}{2}$
 ④ $-\frac{7}{2}$ ⑤ $-\frac{9}{2}$

10. [2 points]

When α and β are solutions of $x^2 - 2x + 6 = 0$,

find $\frac{\beta^2}{\alpha} + \frac{\alpha^2}{\beta}$.

- ① $-\frac{2}{3}$ ② $-\frac{5}{3}$ ③ $-\frac{8}{3}$
④ $-\frac{11}{3}$ ⑤ $-\frac{14}{3}$

11. [2 points]

When $x + y = 3$ and $x^2 + y^2 = 7$, find $x^3 + y^3$.

- ① 6 ② 9 ③ 12 ④ 15 ⑤ 18

12. [2 points]

Find the sum of all real solutions of

$$4^x - 3 \cdot 2^x + \frac{12}{2^x} = 4.$$

- ① $\log_2 3$ ② $\log_2 5$ ③ $\log_2 6$
④ $\log_2 8$ ⑤ $\log_2 10$

13. [2 points]

Find $\log_2 \frac{3}{2} + \log_2 \frac{4}{3} + \log_2 \frac{5}{4} + \dots + \log_2 \frac{32}{31}$.

- ① 2 ② 3 ③ 4 ④ 5 ⑤ 6

14. [2 points]

Find the maximum value of

$$f(x) = 2\sqrt{2} \sin x + 2\sqrt{2} \cos x + 3 \cos \left(x + \frac{\pi}{4}\right).$$

- ① 4 ② 5 ③ 6 ④ 7 ⑤ 8

15. [2 points]

When $A = \begin{pmatrix} 2 & -1 \\ 1 & -1 \end{pmatrix}$ and $A^2 + A^{-1} = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$,

find $a + b + c + d$.

- ① 2 ② 4 ③ 6 ④ 8 ⑤ 10

16. [2 points]

Find the minimum value of a for a positive constant $a > 0$ such that two graphs

$y = \frac{x^2 + 4}{x}$ and $y = a$ meet.

- ① 3 ② 4 ③ 5 ④ 6 ⑤ 7

17. [2 points]

When $2^x = 3^y = 36^z$, find $\frac{z}{x} + \frac{z}{y}$.

- ① $\frac{1}{2}$ ② $\frac{1}{3}$ ③ $\frac{1}{4}$ ④ $\frac{1}{6}$ ⑤ $\frac{1}{12}$

18. [2 points]

When $z = \frac{\sqrt{2 + \sqrt{2}}}{2} - \frac{i\sqrt{2 - \sqrt{2}}}{2}$, find $z^8 + z^4$.

- ① 0 ② 1 ③ i ④ $1 + i$ ⑤ $-1 - i$

19. [2 points]

When ω is a root of $x^2 + x + 1 = 0$, find

$\omega^{2026} + \omega^{20}$.

- ① -1 ② $-i$ ③ $-1 - i$
④ $-2 - i$ ⑤ $-2 - 2i$

20. [2 points]

When $\omega = \frac{\sqrt{3} - i}{2}$, find $\frac{\omega^{12} + 1}{\omega^3 + 1}$.

- ① 1 ② -1 ③ i ④ $1 + i$ ⑤ $1 - i$

21. [2 points]

Find $\sin \frac{\pi}{16} \cos \frac{\pi}{16}$.

- ① $\frac{\sqrt{2-\sqrt{2}}}{3}$ ② $\frac{\sqrt{2-\sqrt{2}}}{4}$ ③ $\frac{\sqrt{2-\sqrt{2}}}{5}$
④ $\frac{\sqrt{2-\sqrt{2}}}{6}$ ⑤ $\frac{\sqrt{2-\sqrt{2}}}{7}$

22. [2 points]

Find $\lim_{x \rightarrow 0} \frac{x^2 - \sin(2x^2)}{1 - \cos x}$.

- ① $-\frac{2}{3}$ ② $-\frac{1}{2}$ ③ -1
④ $-\frac{3}{2}$ ⑤ -2

23. [2 points]

When $\lim_{x \rightarrow 2} \frac{x^2 - 4}{x^2 + ax + b} = \frac{4}{5}$, find $a + b$.

- ① -1 ② -2 ③ -3 ④ -4 ⑤ -5

24. [2 points]

When $\sin \theta = \frac{1}{\sqrt{3}}$ ($\frac{\pi}{2} < \theta < \pi$),

find $\sin 2\theta + \cos 2\theta$.

- ① $\frac{1-\sqrt{2}}{3}$ ② $\frac{1-2\sqrt{2}}{3}$ ③ $\frac{1-3\sqrt{2}}{3}$
④ $\frac{1-4\sqrt{2}}{3}$ ⑤ $\frac{1-5\sqrt{2}}{3}$

25. [2 points]

Find the sum of all solutions to

$3 \cos 2\theta + 10 \cos \theta = 1$ for $0 \leq \theta < 2\pi$.

- ① 0 ② π ③ 2π ④ 3π ⑤ 4π

26. [3 points]

When $A = \begin{pmatrix} 1 & 2 \\ 2 & 3 \end{pmatrix}$, $B = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ and

$ABA^{-1} = \begin{pmatrix} 1 & 0 \\ 2 & 1 \end{pmatrix}$, find $a + b + c + d$.

- ① 2 ② 4 ③ 6 ④ 8 ⑤ 10

27. [3 points]

Find the minimum value of

$f(x) = \frac{3}{2}x^4 + 2x^3 + 3x^2 - 18x + 4$.

- ① $-\frac{11}{2}$ ② $-\frac{13}{2}$ ③ $-\frac{15}{2}$
④ $-\frac{17}{2}$ ⑤ $-\frac{19}{2}$

28. [3 points]

When $z = \frac{1+i}{1-i}$, find $\frac{z}{1+z+z^2+\dots+z^{2025}}$.

- ① i ② $-i$ ③ $\frac{1}{2}$ ④ $\frac{1-i}{2}$ ⑤ $\frac{1+i}{2}$

29. [3 points]

When $f(x) = \sin\left(\frac{3+2x-3x^2}{3}\right)$ ($0 \leq x \leq 1$) has

the maximum value $M = \sin(a)$ at $x = b$, find $a + b$.

- ① $\frac{10}{9}$ ② $\frac{13}{9}$ ③ $\frac{16}{9}$ ④ $\frac{19}{9}$ ⑤ $\frac{22}{9}$

30. [3 points]

Find $\lim_{x \rightarrow \infty} (\sqrt{2x^2 + 3x + 7} - \sqrt{2x^2 + x - 4})$.

- ① $\frac{1}{\sqrt{3}}$ ② $\frac{2}{\sqrt{3}}$ ③ $\frac{1}{\sqrt{2}}$ ④ 1 ⑤ $\sqrt{2}$

31. [3 points]

Find $\int_{-1}^3 (x^2 - 4x - 1) dx$.

- ① -10 ② $-\frac{32}{3}$ ③ $-\frac{34}{3}$
④ -12 ⑤ $-\frac{38}{32}$

32. [3 points]

Find $\int_0^1 (2x-1)^8 dx$.

- ① $\frac{1}{3}$ ② $\frac{1}{4}$ ③ $\frac{1}{6}$ ④ $\frac{1}{8}$ ⑤ $\frac{1}{9}$

33. [3 points]

When M and m are the maximum and minimum values of $f(x) = \frac{1}{3}x^3 - 4x + 7$, ($-1 \leq x \leq 3$), find $M + m$.

- ① $\frac{31}{3}$ ② 11 ③ $\frac{35}{3}$ ④ $\frac{37}{3}$ ⑤ 13

34. [3 points]

When $y = ax + b$ is the tangent line to

$y = \frac{-2x+7}{x^2+x+3}$ at $x = -1$, find $a + b$.

- ① $\frac{11}{3}$ ② $\frac{13}{3}$ ③ 5 ④ $\frac{17}{3}$ ⑤ $\frac{19}{3}$

35. [3 points]

When $f(x) = \frac{\sqrt{2x+5}}{x^2-x+7}$, find $f'(2)$.

- ① $-\frac{2}{27}$ ② $-\frac{4}{27}$ ③ $-\frac{2}{9}$ ④ $-\frac{8}{27}$ ⑤ $-\frac{10}{27}$

36. [4 points]

When $f(x)$ satisfies

$f(x) = \frac{5}{3}x^3 - 6x^2 + 9x - \int_0^x f'(2t-3) dt$,

find $f'(3)$.

- ① 1 ② 3 ③ 5 ④ 7 ⑤ 9

37. [4 points]

When $\int_1^x tf(t) dt = \frac{4}{3}x^3 - 2x^2 + ax + b$ and

$f(2) = 5$ for a continuous function $f(x)$, find ab .

- ① $-\frac{2}{3}$ ② $-\frac{4}{3}$ ③ -2 ④ $-\frac{8}{3}$ ⑤ $-\frac{10}{3}$

38. [4 points]

Find $\int_0^1 \frac{2x^2+1}{\sqrt{2x^3+3x+4}} dx$.

- ① 1 ② $\frac{2}{3}$ ③ $\frac{4}{3}$ ④ 2 ⑤ $\frac{8}{3}$

39. [4 points]

When

$c_n = \frac{1}{\sqrt{n^2+n}} + \frac{1}{\sqrt{n^2+2n}} + \dots + \frac{1}{\sqrt{n^2+n^2}}$, find

$\lim_{n \rightarrow \infty} c_n$.

- ① $\sqrt{2}-1$ ② $\sqrt{2}-2$ ③ $2\sqrt{2}-1$
④ $2\sqrt{2}-2$ ⑤ $3\sqrt{2}-1$

40. [4 points]

Find the region of the area enclosed by

$x^3 - 3x^2 + 2x - 5 = 0$ and $x^3 - 4x^2 + 3x + 1 = 0$

- ① $\frac{121}{6}$ ② $\frac{41}{2}$ ③ $\frac{125}{6}$ ④ $\frac{127}{6}$ ⑤ $\frac{43}{2}$

2025 IUT 2nd Admission Test(SBL Scholarship)

Math Examination(TYPE D)

< Multiple choice Types > There is only one correct answer for each question. Mark your choice on the OMR answer sheet.

- The points for each question are listed next to the question number.
- You can use the right side of each page for your memo.

1. [2 points]

Simplify $\sqrt{2} \times \sqrt[3]{4} \div \sqrt[4]{8}$.

- ① $\sqrt[4]{2}$ ② $\sqrt[3]{2}$ ③ $\sqrt[12]{2^5}$ ④ $\sqrt{2}$ ⑤ $\sqrt[12]{2^7}$

2. [2 points]

When $x^{\frac{1}{3}} + x^{-\frac{1}{3}} = 3$, find $x^2 + x^{-2}$.

- ① 316 ② 318 ③ 320 ④ 322 ⑤ 324

3. [2 points]

Simplify $\frac{\sqrt{2} - \sqrt{3} - \sqrt{4} + \sqrt{6}}{\sqrt{2} + \sqrt{3} - \sqrt{4} - \sqrt{6}}$.

- ① $2\sqrt{6} - 1$ ② $2\sqrt{6} - 2$ ③ $2\sqrt{6} - 3$
 ④ $2\sqrt{6} - 4$ ⑤ $2\sqrt{6} - 5$

4. [2 points]

Simplify $\frac{1}{\sqrt{7+2\sqrt{12}}}$.

- ① $2 - \sqrt{3}$ ② $2 - 2\sqrt{3}$ ③ 2
 ④ $2 + \sqrt{3}$ ⑤ $2 + 2\sqrt{3}$

5. [2 points]

Find $\frac{1}{(1-\sqrt{2})^2} - \frac{1}{(1+\sqrt{2})^2}$.

- ① $2\sqrt{2}$ ② $4\sqrt{2}$ ③ $6\sqrt{2}$
 ④ $8\sqrt{2}$ ⑤ $10\sqrt{2}$

6. [2 points]

When α and β are solutions of $x^2 - 2x + 6 = 0$, find $\frac{\beta^2}{\alpha} + \frac{\alpha^2}{\beta}$.

- ① $-\frac{2}{3}$ ② $-\frac{5}{3}$ ③ $-\frac{8}{3}$
 ④ $-\frac{11}{3}$ ⑤ $-\frac{14}{3}$

7. [2 points]

When $42^x = 27$ and $14^y = \frac{1}{81}$, find $\frac{3}{2x} + \frac{2}{y}$.

- ① $\frac{1}{2}$ ② $\frac{1}{3}$ ③ $\frac{1}{6}$ ④ $\frac{1}{8}$ ⑤ $\frac{1}{9}$

8. [2 points]

Find $\sum_{n=2}^{10} \frac{1}{2^n}$.

- ① $\frac{63}{128}$ ② $\frac{127}{256}$ ③ $\frac{255}{512}$ ④ $\frac{511}{1024}$ ⑤ $\frac{1023}{2048}$

9. [2 points]

Simplify $\frac{1}{(a+1)(a^2+1)(a^4+1)(a^8+1)}$ when $a = \sqrt[8]{3}$.

- ① $\frac{\sqrt[8]{3}-1}{2}$ ② $\frac{\sqrt[8]{3}-1}{4}$ ③ $\frac{\sqrt[8]{3}-1}{6}$
 ④ $\frac{\sqrt[8]{3}-1}{8}$ ⑤ $\frac{\sqrt[8]{3}-1}{10}$

10. [2 points]

When α, β, γ are the solutions of $x^3 - 3x^2 - 5x + 6 = 0$, find

$\frac{\beta+\gamma}{\alpha} + \frac{\gamma+\alpha}{\beta} + \frac{\alpha+\beta}{\gamma}$.

- ① $-\frac{1}{2}$ ② $-\frac{3}{2}$ ③ $-\frac{5}{2}$
 ④ $-\frac{7}{2}$ ⑤ $-\frac{9}{2}$

11. [2 points]

When $A = \begin{pmatrix} 2 & -1 \\ 1 & -1 \end{pmatrix}$ and $A^2 + A^{-1} = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$,

find $a + b + c + d$.

- ① 2 ② 4 ③ 6 ④ 8 ⑤ 10

12. [2 points]

When $x + y = 3$ and $x^2 + y^2 = 7$, find $x^3 + y^3$.

- ① 6 ② 9 ③ 12 ④ 15 ⑤ 18

13. [2 points]

Find the sum of all real solutions of

$$4^x - 3 \cdot 2^x + \frac{12}{2^x} = 4.$$

- ① $\log_2 3$ ② $\log_2 5$ ③ $\log_2 6$
④ $\log_2 8$ ⑤ $\log_2 10$

14. [2 points]

Find $\log_2 \frac{3}{2} + \log_2 \frac{4}{3} + \log_2 \frac{5}{4} + \dots + \log_2 \frac{32}{31}$.

- ① 2 ② 3 ③ 4 ④ 5 ⑤ 6

15. [2 points]

Find the maximum value of

$$f(x) = 2\sqrt{2} \sin x + 2\sqrt{2} \cos x + 3 \cos \left(x + \frac{\pi}{4} \right).$$

- ① 4 ② 5 ③ 6 ④ 7 ⑤ 8

16. [2 points]

When $\omega = \frac{\sqrt{3} - i}{2}$, find $\frac{\omega^{12} + 1}{\omega^3 + 1}$.

- ① 1 ② -1 ③ i ④ $1 + i$ ⑤ $1 - i$

17. [2 points]

Find the minimum value of a for a positive constant $a > 0$ such that two graphs

$$y = \frac{x^2 + 4}{x} \text{ and } y = a \text{ meet.}$$

- ① 3 ② 4 ③ 5 ④ 6 ⑤ 7

18. [2 points]

When $2^x = 3^y = 36^z$, find $\frac{z}{x} + \frac{z}{y}$.

- ① $\frac{1}{2}$ ② $\frac{1}{3}$ ③ $\frac{1}{4}$ ④ $\frac{1}{6}$ ⑤ $\frac{1}{12}$

19. [2 points]

When $z = \frac{\sqrt{2 + \sqrt{2}}}{2} - \frac{i\sqrt{2 - \sqrt{2}}}{2}$, find $z^8 + z^4$.

- ① 0 ② 1 ③ i ④ $1 + i$ ⑤ $-1 - i$

20. [2 points]

When ω is a root of $x^2 + x + 1 = 0$, find

$$\omega^{2026} + \omega^{20}.$$

- ① -1 ② $-i$ ③ $-1 - i$
④ $-2 - i$ ⑤ $-2 - 2i$

21. [2 points]

Find the sum of all solutions to

$$3 \cos 2\theta + 10 \cos \theta = 1 \text{ for } 0 \leq \theta < 2\pi.$$

- ① 0 ② π ③ 2π ④ 3π ⑤ 4π

22. [2 points]

Find $\sin \frac{\pi}{16} \cos \frac{\pi}{16}$.

- ① $\frac{\sqrt{2-\sqrt{2}}}{3}$ ② $\frac{\sqrt{2-\sqrt{2}}}{4}$ ③ $\frac{\sqrt{2-\sqrt{2}}}{5}$
④ $\frac{\sqrt{2-\sqrt{2}}}{6}$ ⑤ $\frac{\sqrt{2-\sqrt{2}}}{7}$

23. [2 points]

Find $\lim_{x \rightarrow 0} \frac{x^2 - \sin(2x^2)}{1 - \cos x}$.

- ① $-\frac{2}{3}$ ② $-\frac{1}{2}$ ③ -1
④ $-\frac{3}{2}$ ⑤ -2

24. [2 points]

When $\lim_{x \rightarrow 2} \frac{x^2 - 4}{x^2 + ax + b} = \frac{4}{5}$, find $a + b$.

- ① -1 ② -2 ③ -3 ④ -4 ⑤ -5

25. [2 points]

When $\sin \theta = \frac{1}{\sqrt{3}}$ ($\frac{\pi}{2} < \theta < \pi$),

find $\sin 2\theta + \cos 2\theta$.

- ① $\frac{1-\sqrt{2}}{3}$ ② $\frac{1-2\sqrt{2}}{3}$ ③ $\frac{1-3\sqrt{2}}{3}$
④ $\frac{1-4\sqrt{2}}{3}$ ⑤ $\frac{1-5\sqrt{2}}{3}$

26. [3 points]

Find $\lim_{x \rightarrow \infty} (\sqrt{2x^2 + 3x + 7} - \sqrt{2x^2 + x - 4})$.

- ① $\frac{1}{\sqrt{3}}$ ② $\frac{2}{\sqrt{3}}$ ③ $\frac{1}{\sqrt{2}}$ ④ 1 ⑤ $\sqrt{2}$

27. [3 points]

When $A = \begin{pmatrix} 1 & 2 \\ 2 & 3 \end{pmatrix}$, $B = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ and

$ABA^{-1} = \begin{pmatrix} 1 & 0 \\ 2 & 1 \end{pmatrix}$, find $a + b + c + d$.

- ① 2 ② 4 ③ 6 ④ 8 ⑤ 10

28. [3 points]

Find the minimum value of

$f(x) = \frac{3}{2}x^4 + 2x^3 + 3x^2 - 18x + 4$.

- ① $-\frac{11}{2}$ ② $-\frac{13}{2}$ ③ $-\frac{15}{2}$
④ $-\frac{17}{2}$ ⑤ $-\frac{19}{2}$

29. [3 points]

When $z = \frac{1+i}{1-i}$, find $\frac{z}{1+z+z^2+\dots+z^{2025}}$.

- ① i ② $-i$ ③ $\frac{1}{2}$ ④ $\frac{1-i}{2}$ ⑤ $\frac{1+i}{2}$

30. [3 points]

When $f(x) = \sin\left(\frac{3+2x-3x^2}{3}\right)$ ($0 \leq x \leq 1$) has

the maximum value $M = \sin(a)$ at $x = b$, find $a + b$.

- ① $\frac{10}{9}$ ② $\frac{13}{9}$ ③ $\frac{16}{9}$ ④ $\frac{19}{9}$ ⑤ $\frac{22}{9}$

31. [3 points]

When $f(x) = \frac{\sqrt{2x+5}}{x^2-x+7}$, find $f'(2)$.

- ① $-\frac{2}{27}$ ② $-\frac{4}{27}$ ③ $-\frac{2}{9}$ ④ $-\frac{8}{27}$ ⑤ $-\frac{10}{27}$

32. [3 points]

Find $\int_{-1}^3 (x^2 - 4x - 1) dx$.

- ① -10 ② $-\frac{32}{3}$ ③ $-\frac{34}{3}$
④ -12 ⑤ $-\frac{38}{32}$

33. [3 points]

Find $\int_0^1 (2x-1)^8 dx$.

- ① $\frac{1}{3}$ ② $\frac{1}{4}$ ③ $\frac{1}{6}$ ④ $\frac{1}{8}$ ⑤ $\frac{1}{9}$

34. [3 points]

When M and m are the maximum and minimum values of $f(x) = \frac{1}{3}x^3 - 4x + 7$, ($-1 \leq x \leq 3$), find $M + m$.

- ① $\frac{31}{3}$ ② 11 ③ $\frac{35}{3}$ ④ $\frac{37}{3}$ ⑤ 13

35. [3 points]

When $y = ax + b$ is the tangent line to

$y = \frac{-2x+7}{x^2+x+3}$ at $x = -1$, find $a + b$.

- ① $\frac{11}{3}$ ② $\frac{13}{3}$ ③ 5 ④ $\frac{17}{3}$ ⑤ $\frac{19}{3}$

36. [4 points]

Find the region of the area enclosed by

$x^3 - 3x^2 + 2x - 5 = 0$ and $x^3 - 4x^2 + 3x + 1 = 0$

- ① $\frac{121}{6}$ ② $\frac{41}{2}$ ③ $\frac{125}{6}$ ④ $\frac{127}{6}$ ⑤ $\frac{43}{2}$

37. [4 points]

When $f(x)$ satisfies

$f(x) = \frac{5}{3}x^3 - 6x^2 + 9x - \int_0^x f'(2t-3) dt$,

find $f'(3)$.

- ① 1 ② 3 ③ 5 ④ 7 ⑤ 9

38. [4 points]

When $\int_1^x tf(t) dt = \frac{4}{3}x^3 - 2x^2 + ax + b$ and

$f(2) = 5$ for a continuous function $f(x)$, find ab .

- ① $-\frac{2}{3}$ ② $-\frac{4}{3}$ ③ -2 ④ $-\frac{8}{3}$ ⑤ $-\frac{10}{3}$

39. [4 points]

Find $\int_0^1 \frac{2x^2+1}{\sqrt{2x^3+3x+4}} dx$.

- ① 1 ② $\frac{2}{3}$ ③ $\frac{4}{3}$ ④ 2 ⑤ $\frac{8}{3}$

40. [4 points]

When

$c_n = \frac{1}{\sqrt{n^2+n}} + \frac{1}{\sqrt{n^2+2n}} + \dots + \frac{1}{\sqrt{n^2+n^2}}$, find

$\lim_{n \rightarrow \infty} c_n$.

- ① $\sqrt{2}-1$ ② $\sqrt{2}-2$ ③ $2\sqrt{2}-1$
④ $2\sqrt{2}-2$ ⑤ $3\sqrt{2}-1$

2025 IUT 2nd SBL Scholar Answer Sheets

[TypeA]

No.	1	2	3	4	5	6	7	8	9	10
Ans.	①	②	③	④	⑤	④	①	⑤	①	④
No.	11	12	13	14	15	16	17	18	19	20
Ans.	③	②	①	⑤	③	⑤	①	④	②	①
No.	21	22	23	24	25	26	27	28	29	30
Ans.	⑤	②	③	②	⑤	⑤	②	③	④	③
No.	31	32	33	34	35	36	37	38	39	40
Ans.	④	①	①	②	⑤	②	④	③	⑤	④

[TypeB]

No.	1	2	3	4	5	6	7	8	9	10
Ans.	⑤	①	②	③	④	④	④	①	⑤	①
No.	11	12	13	14	15	16	17	18	19	20
Ans.	③	③	②	①	⑤	①	⑤	①	④	②
No.	21	22	23	24	25	26	27	28	29	30
Ans.	⑤	⑤	②	③	②	③	⑤	②	③	④
No.	31	32	33	34	35	36	37	38	39	40
Ans.	⑤	④	①	①	②	④	②	④	③	⑤

[TypeC]

No.	1	2	3	4	5	6	7	8	9	10
Ans.	④	⑤	①	②	③	①	④	④	①	⑤
No.	11	12	13	14	15	16	17	18	19	20
Ans.	⑤	③	③	②	①	②	①	⑤	①	④
No.	21	22	23	24	25	26	27	28	29	30
Ans.	②	⑤	⑤	②	③	④	③	⑤	②	③
No.	31	32	33	34	35	36	37	38	39	40
Ans.	②	⑤	④	①	①	⑤	④	②	④	③

[TypeD]

No.	1	2	3	4	5	6	7	8	9	10
Ans.	③	④	⑤	①	②	⑤	①	④	④	①
No.	11	12	13	14	15	16	17	18	19	20
Ans.	①	⑤	③	③	②	④	②	①	⑤	①
No.	21	22	23	24	25	26	27	28	29	30
Ans.	③	②	⑤	⑤	②	③	④	③	⑤	②
No.	31	32	33	34	35	36	37	38	39	40
Ans.	①	②	⑤	④	①	③	⑤	④	②	④

2025 IUT 2nd Admission Test(SBL Scholarship) Solution

(1) Find $\frac{1}{\sqrt{7+2\sqrt{12}}}$.

(SOL) $\frac{1}{\sqrt{7+4\sqrt{3}}} = \frac{1}{\sqrt{(\sqrt{4}+\sqrt{3})^2}} = \frac{1}{2+\sqrt{3}} = \frac{1}{2+\sqrt{3}} \times \frac{(2-\sqrt{3})}{(2-\sqrt{3})} = 2-\sqrt{3}$.

(2) Find $\frac{1}{(1-\sqrt{2})^2} - \frac{1}{(1+\sqrt{2})^2}$.

(SOL) $\frac{1}{(1-\sqrt{2})^2} - \frac{1}{(1+\sqrt{2})^2} = \frac{(1+\sqrt{2})^2 - (1-\sqrt{2})^2}{(1-\sqrt{2})^2(1+\sqrt{2})^2} = \frac{4\sqrt{2}}{(1-2)^2} = 4\sqrt{2}$.

(3) Simplify $\sqrt{2} \times \sqrt[3]{4} \div \sqrt[4]{8}$.

(SOL) $\sqrt{2} \times \sqrt[3]{4} \div \sqrt[4]{8} = 2^{\frac{1}{2}} 2^{\frac{2}{3}} 2^{-\frac{3}{4}} = 2^{\frac{6+8-9}{12}} = 2^{\frac{5}{12}} = \sqrt[12]{2^5}$.

(4) When $x^{\frac{1}{3}} + x^{-\frac{1}{3}} = 3$, find $x^2 + x^{-2}$.

(SOL) Since $(x^{\frac{1}{3}} + x^{-\frac{1}{3}})^3 = x + x^{-1} + 3(x^{\frac{1}{3}} + x^{-\frac{1}{3}}) = x + x^{-1} + 9 = 3^3$, we have $x + x^{-1} = 18$. It follows that $x^2 + x^{-2} = (x + x^{-1})^2 - 2 = 18^2 - 2 = 322$.

(5) Simplify $\frac{\sqrt{2}-\sqrt{3}-\sqrt{4}+\sqrt{6}}{\sqrt{2}+\sqrt{3}-\sqrt{4}-\sqrt{6}}$.

(SOL) $\frac{\sqrt{2}-\sqrt{3}-\sqrt{4}+\sqrt{6}}{\sqrt{2}+\sqrt{3}-\sqrt{4}-\sqrt{6}} = \frac{(\sqrt{2}-\sqrt{3})(1-\sqrt{2})}{(\sqrt{2}+\sqrt{3})(1-\sqrt{2})} = \frac{(\sqrt{2}-\sqrt{3})}{(\sqrt{2}+\sqrt{3})} \times \frac{(\sqrt{3}-\sqrt{2})}{(\sqrt{3}-\sqrt{2})} = 2\sqrt{6}-5$.

(6) When $a = \sqrt[8]{3}$, find $\frac{1}{(a+1)(a^2+1)(a^4+1)(a^8+1)}$.

(SOL) $\frac{1}{(a+1)(a^2+1)(a^4+1)(a^8+1)} \times \frac{a-1}{a-1} = \frac{a-1}{(a^8-1)(a^8+1)} = \frac{\sqrt[8]{3}-1}{(3-1)(3+1)} = \frac{\sqrt[8]{3}-1}{8}$.

(7) When α, β, γ are the solutions of $x^3 - 3x^2 - 5x + 6 = 0$, find $\frac{\beta+\gamma}{\alpha} + \frac{\gamma+\alpha}{\beta} + \frac{\alpha+\beta}{\gamma}$.

(SOL) Note that $\alpha + \beta + \gamma = 3$, $\alpha\beta + \beta\gamma + \gamma\alpha = -5$, $\alpha\beta\gamma = -6$. Since $\alpha + \beta + \gamma = 3$, it follows that $\alpha + \beta = 3 - \gamma$, $\beta + \gamma = 3 - \alpha$, $\gamma + \alpha = 3 - \beta$. Hence, it follows that

$$\begin{aligned} \frac{\beta+\gamma}{\alpha} + \frac{\gamma+\alpha}{\beta} + \frac{\alpha+\beta}{\gamma} &= \frac{3-\alpha}{\alpha} + \frac{3-\beta}{\beta} + \frac{3-\gamma}{\gamma} \\ &= 3 \cdot \left(\frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\gamma} \right) - 3 = 3 \cdot \left(\frac{\alpha\beta + \beta\gamma + \gamma\alpha}{\alpha\beta\gamma} \right) - 3 = 3 \cdot \frac{-5}{-6} - 3 = \frac{5}{2} - 3 = -\frac{1}{2}. \end{aligned}$$

(8) When α and β are solutions of $x^2 - 2x + 6 = 0$, find $\frac{\beta^2}{\alpha} + \frac{\alpha^2}{\beta}$.

(SOL) Since $\alpha + \beta = 2$, $\alpha\beta = 6$, it follows that

$$\frac{\beta^2}{\alpha} + \frac{\alpha^2}{\beta} = \frac{\alpha^3 + \beta^3}{\alpha\beta} = \frac{(\alpha + \beta)^3 - 3\alpha\beta(\alpha + \beta)}{\alpha\beta} = -\frac{14}{3}.$$

(9) When $42^x = 27$ and $14^y = \frac{1}{81}$, find $\frac{3}{2x} + \frac{2}{y}$.

(SOL) Note that $x = \log_{42} 27 = 3\log_{42} 3$, $y = \log_{14} 81^{-1} = -4\log_{14} 3$. It follows that

$$\frac{3}{2x} + \frac{2}{y} = \frac{1}{2} \log_3 42 + \frac{1}{-2} \log_3 14 = \frac{1}{2} \log_3 3 = \frac{1}{2}.$$

(10) Find $\sum_{n=2}^{10} \frac{1}{2^n}$.

$$(SOL) \sum_{n=2}^{10} \frac{1}{2^n} = \frac{\frac{1}{2^2} \left(1 - \frac{1}{2^9}\right)}{1 - \frac{1}{2}} = \frac{1}{2} - \frac{1}{2^{10}} = \frac{511}{1024}.$$

(11) Find $\log_2 \frac{3}{2} + \log_2 \frac{4}{3} + \log_2 \frac{5}{4} + \dots + \log_2 \frac{32}{31}$.

$$(SOL) \log_2 \frac{3}{2} + \log_2 \frac{4}{3} + \log_2 \frac{5}{4} + \dots + \log_2 \frac{32}{31} = \log_2 \left(\frac{3}{2} \cdot \frac{4}{3} \cdot \frac{5}{4} \cdots \frac{32}{31} \right) = \log_2 (16) = 4.$$

(12) Find the maximum value M of $f(x) = 2\sqrt{2} \sin x + 2\sqrt{2} \cos x + 3 \cos \left(x + \frac{\pi}{4}\right)$.

(SOL) Note that

$$f(x) = 2\sqrt{2}(\sin x + \cos x) + 3 \cos \left(x + \frac{\pi}{4}\right) = 4 \sin \left(x + \frac{\pi}{4}\right) + 3 \cos \left(x + \frac{\pi}{4}\right) = 5 \sin(x + \alpha) \text{ for some } \alpha.$$

It follows that $M = 5$.

(13) When $A = \begin{pmatrix} 2 & -1 \\ 1 & -1 \end{pmatrix}$ and $A^2 + A^{-1} = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$, find $a + b + c + d$.

(SOL) Note that $A^2 + A^{-1} = \begin{pmatrix} 2 & -1 \\ 1 & -1 \end{pmatrix} \begin{pmatrix} 2 & -1 \\ 1 & -1 \end{pmatrix} + \frac{1}{-1} \begin{pmatrix} -1 & 1 \\ -1 & 2 \end{pmatrix} = \begin{pmatrix} 3 & -1 \\ 1 & 0 \end{pmatrix} + \begin{pmatrix} 1 & -1 \\ 1 & -2 \end{pmatrix} = \begin{pmatrix} 4 & -2 \\ 2 & -2 \end{pmatrix}$. It follows that $a + b + c + d = 2$.

(14) When $x + y = 3$ and $x^2 + y^2 = 7$, find $x^3 + y^3$.

(SOL) Since $xy = \frac{(x+y)^2 - (x^2 + y^2)}{2} = 1$, it follows that $x^3 + y^3 = (x+y)^3 - 3xy(x+y) = 18$.

(15) Find the sum of all real solutions of $4^x - 3 \cdot 2^x + \frac{12}{2^x} = 4$.

(SOL) Since $(2^x)^3 - 3(2^x)^2 - 4(2^x) + 12 = (2^x + 2)(2^x - 2)(2^x - 3)$, it follows that the sum is $1 + \log_2 3 = \log_2 2 + \log_2 3 = \log_2 6$.

(16) When $z = \frac{\sqrt{2+\sqrt{2}}}{2} - \frac{i\sqrt{2-\sqrt{2}}}{2}$, find $z^8 + z^4$.

(SOL) Since $z^2 = \frac{1-i}{\sqrt{2}}$ and $z^4 = -i$, it follows that $z^8 + z^4 = -1 - i$.

(17) When ω is a root of $x^2 + x + 1 = 0$, find $\omega^{2026} + \omega^{20}$.

(SOL) Since $\omega^3 = 1$, it follows that $\omega^{2026} + \omega^{20} = (\omega^3)^{675} \omega + (\omega^3)^6 \omega^2 = \omega + \omega^2 = -1$.

(18) When $\omega = \frac{\sqrt{3}-i}{2}$, find $\frac{\omega^{12}+1}{\omega^3+1}$.

(SOL) Note that $\omega^3 = \frac{3\sqrt{3}+i-3\sqrt{3}i(\sqrt{3}-i)}{8} = -i$. It follows that

$$\frac{\omega^{12}+1}{\omega^3+1} = \frac{(-i)^4+1}{-i+1} = \frac{2}{1-i} \times \frac{1+i}{1+i} = 1+i.$$

(19) Find the minimum value of a for positive constant $a > 0$ such that two graphs

$y = \frac{x^2+4}{x}$ and $y = a$ meet.

(SOL) Since $\frac{x^2+4}{x} = a$, that is, $(x^2+4) - ax = 0$ has a real solution, $D = a^2 - 4(4) \geq 0$. Since $a > 0$, $a \geq 4$. Thus, the minimum value of a is 4.

(20) When $2^x = 3^y = 36^z$, find $\frac{z}{x} + \frac{z}{y}$.

(SOL) $\frac{z}{x} + \frac{z}{y} = \log_{36} 2 + \log_{36} 3 = \log_{36} 6 = \log_{6^2} 6 = \frac{1}{2}$.

(21) When $\lim_{x \rightarrow 2} \frac{x^2-4}{x^2+ax+b} = \frac{4}{5}$, find $a+b$.

(SOL) Since $2^2 + 2a + b = 0$, $\lim_{x \rightarrow 2} \frac{x^2-4}{x^2+ax+b} = \lim_{x \rightarrow 2} \frac{(x-2)(x+2)}{(x-2)(x-\frac{b}{2})} = \frac{4}{5}$, we get $b = -6$ and $a = 1$.

Hence, $a+b = -5$.

(22) When $\sin \theta = \frac{1}{\sqrt{3}}$ ($\frac{\pi}{2} < \theta < \pi$), find $\sin 2\theta + \cos 2\theta$.

(SOL) Note that $\cos \theta = -\frac{\sqrt{2}}{\sqrt{3}}$. It follows that

$$\sin 2\theta + \cos 2\theta = 2\sin \theta \cos \theta + (1 - 2\sin^2 \theta) = -\frac{2\sqrt{2}}{3} + 1 - \frac{2}{3} = \frac{1-2\sqrt{2}}{3}.$$

(23) Find the sum of all solutions to $3\cos 2\theta + 10\cos \theta = 1$ for $0 \leq \theta < 2\pi$.

(SOL) Note that $3(2\cos^2 \theta - 1) + 10\cos \theta - 1 = 6\cos^2 \theta + 10\cos \theta - 4 = 2(3\cos^2 \theta + 5\cos \theta - 2) = 0$, that is, $(3\cos \theta - 1)(\cos \theta + 2) = 0$. Since $|\cos \theta| \leq 1$, $\cos \theta = \frac{1}{3}$. Note that $\cos(2\pi - \theta) = \cos \theta$. Hence, the sum of all solutions is $\theta + (2\pi - \theta) = 2\pi$.

(24) Find $\sin \frac{\pi}{16} \cos \frac{\pi}{16}$.

(SOL) Since $\sin \frac{\pi}{16} \cos \frac{\pi}{16} = \frac{1}{2} \sin \frac{\pi}{8}$,

we have $\left(\frac{1}{2} \sin \frac{\pi}{8}\right)^2 = \frac{1}{4} \frac{1 - \cos \frac{\pi}{4}}{2} = \frac{1}{4} \frac{1 - \frac{\sqrt{2}}{2}}{2} = \frac{1}{4} \frac{2 - \sqrt{2}}{4}$. It follows that

$$\sin \frac{\pi}{16} \cos \frac{\pi}{16} = \frac{\sqrt{2 - \sqrt{2}}}{4}.$$

(25) Find $\lim_{x \rightarrow 0} \frac{x^2 - \sin(2x^2)}{1 - \cos x}$.

(SOL)

Note that $\frac{x^2 - \sin(2x^2)}{1 - \cos x} = \left(\frac{x^2}{2x^2} - \frac{\sin(2x^2)}{2x^2}\right) \frac{2x^2}{1 - \cos x} \frac{1 + \cos x}{1 + \cos x} = \left(\frac{1}{2} - \frac{\sin(2x^2)}{2x^2}\right) \frac{2x^2}{\sin^2 x} \frac{1 + \cos x}{1}$

It follows that $\lim_{x \rightarrow 0} \frac{x^2 - \sin(2x^2)}{1 - \cos x} = \lim_{x \rightarrow 0} \left(\frac{1}{2} - \frac{\sin(2x^2)}{2x^2}\right) \frac{2x^2}{\sin^2 x} \frac{1 + \cos x}{1} = \left(\frac{1}{2} - 1\right) \frac{2}{1} \frac{2}{1} = -2$.

(26) When $z = \frac{1+i}{1-i}$, find $\frac{z}{1+z+z^2+\dots+z^{2025}}$.

(SOL) Since $z = i$, $z^4 = 1$, it follows that $\frac{z}{1+z+z^2+\dots+z^{2025}} = \frac{z(1-z)}{1-z^{2026}} = \frac{1+i}{2}$.

(27) When $f(x) = \sin\left(\frac{3+2x-3x^2}{3}\right)$ ($0 \leq x \leq 1$) has the maximum value $M = \sin(a)$ at $x = b$, find $a + b$.

(SOL) Note that $f(x) = \sin\left(\frac{3+2x-3x^2}{3}\right) = \sin\left(\frac{10-(3x-1)^2}{9}\right)$.

Let $y := \frac{10-(3x-1)^2}{9}$. Then, since $0 \leq x \leq 1$, $y\left(\frac{1}{3}\right) = \frac{10}{9}$ and $\frac{2}{3} \leq y \leq \frac{10}{9}$. And, since the sine function is increasing on $[0, \frac{\pi}{2}]$, we have $M = \sin\left(\frac{10}{9}\right) = f\left(\frac{1}{3}\right)$, which means

$a = \frac{10}{9}$, $b = \frac{1}{3}$. It follows that $a + b = \frac{10}{9} + \frac{1}{3} = \frac{13}{9}$.

(28) Find $\lim_{x \rightarrow \infty} (\sqrt{2x^2 + 3x + 7} - \sqrt{2x^2 + x - 4})$.

(SOL) Note that $(\sqrt{2x^2 + 3x + 7} - \sqrt{2x^2 + x - 4}) = \frac{2x + 11}{(\sqrt{2x^2 + 3x + 7} + \sqrt{2x^2 + x - 4})}$. It follows

that $\lim_{x \rightarrow \infty} (\sqrt{2x^2 + 3x + 7} - \sqrt{2x^2 + x - 4}) = \lim_{x \rightarrow \infty} \frac{2x + 11}{(\sqrt{2x^2 + 3x + 7} + \sqrt{2x^2 + x - 4})}$

$$= \lim_{x \rightarrow \infty} \frac{2 + \frac{11}{x}}{\sqrt{2 + \frac{3}{x} + \frac{7}{x^2}} + \sqrt{2 + \frac{1}{x} - \frac{4}{x^2}}} = \frac{1}{\sqrt{2}}.$$

(29) When $A = \begin{pmatrix} 1 & 2 \\ 2 & 3 \end{pmatrix}$, $B = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ and $ABA^{-1} = \begin{pmatrix} 1 & 0 \\ 2 & 1 \end{pmatrix}$, find $a+b+c+d$.

(SOL) $B = A^{-1} \begin{pmatrix} 1 & 0 \\ 2 & 1 \end{pmatrix} A = - \begin{pmatrix} 3 & -2 \\ -2 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 2 & 1 \end{pmatrix} \begin{pmatrix} 1 & 2 \\ 2 & 3 \end{pmatrix} = \begin{pmatrix} -3 & 2 \\ 2 & -1 \end{pmatrix} \begin{pmatrix} 1 & 2 \\ 2 & 3 \end{pmatrix} = \begin{pmatrix} 5 & 8 \\ -2 & -3 \end{pmatrix}$. It follows that $a+b+c+d = 8$

(30) Find the minimum value of $f(x) = \frac{3}{2}x^4 + 2x^3 + 3x^2 - 18x + 4$.

(SOL) Since $f'(x) = 6x^3 + 6x^2 + 6x - 18 = 6(x^3 + x^2 + x - 3) = (x^2 + 2x + 3)(x - 1)$, $f(x)$ has a minimum value at $x = 1$. The minimum value is $f(1) = \frac{3}{2} + 2 + 3 - 18 + 4 = -\frac{15}{2}$.

(31) When M and m are the maximum and minimum values of $f(x) = \frac{1}{3}x^3 - 4x + 7$, ($-1 \leq x \leq 3$), find $M + m$.

(SOL) Since $f'(x) = x^2 - 4 = (x + 2)(x - 2)$, $f(x)$ has critical values at $x = -2, 2$. Since $f(-1) = -\frac{1}{3} + 4 + 7 = \frac{32}{3}$, $f(3) = 9 - 12 + 7 = 4$, and $f(2) = \frac{8}{3} - 8 + 7 = \frac{5}{3}$, it follows that $M = \frac{32}{3}$ and $m = \frac{5}{3}$. Hence, $M + m = \frac{37}{3}$.

(32) When $y = ax + b$ is the tangent line to $y = \frac{-2x + 7}{x^2 + x + 3}$ at $x = -1$, find $a + b$.

(SOL) Let $f(x) = \frac{-2x + 7}{x^2 + x + 3}$. Since $f'(x) = \frac{-2(x^2 + x + 3) - (-2x + 7)(2x + 1)}{(x^2 + x + 3)^2}$, it follows that

$f'(-1) = \frac{-6 + 9}{9} = \frac{1}{3}$. Since the line passes $(-1, 3)$, the tangent line is

$y = \frac{1}{3}(x + 1) + 3 = \frac{1}{3}x + \frac{10}{3}$. Hence, $a + b = \frac{11}{3}$.

(33) When $f(x) = \frac{\sqrt{2x + 5}}{x^2 - x + 7}$, find $f'(2)$.

(SOL) Since $f'(x) = \frac{\frac{1}{\sqrt{2x + 5}} \cdot (x^2 - x + 7) - \sqrt{2x + 5}(2x - 1)}{(x^2 - x + 7)^2}$, it follows that

$f'(2) = \frac{\frac{1}{3} \cdot 9 - 3 \cdot 3}{81} = \frac{-6}{81} = -\frac{2}{27}$.

(34) Find $\int_{-1}^3 (x^2 - 4x - 1) dx$.

(SOL) $\int_{-1}^3 (x^2 - 4x - 1) dx = \left[\frac{1}{3}x^3 - 2x^2 - x \right]_{-1}^3 = (9 - 18 - 3) - \left(-\frac{1}{3} - 2 + 1 \right)$
 $= -12 + \frac{4}{3} = -\frac{32}{3}$.

(35) Find $\int_0^1 (2x-1)^8 dx$.

(SOL) Setting $u = 2x-1$, then it follows that $du = 2dx$ and hence

$$\int_0^1 (2x-1)^8 dx = \int_{-1}^1 u^8 \cdot \frac{1}{2} du = \int_0^1 u^8 du = \frac{1}{9}.$$

(36) Find $\int_0^1 \frac{2x^2+1}{\sqrt{2x^3+3x+4}} dx$.

(SOL) Setting $u = 2x^3+3x+4$, it follows that $du = (6x^2+3)dx = 3(2x^2+1)dx$ and

$$\int_0^1 \frac{2x^2+1}{\sqrt{2x^3+3x+4}} dx = \int_4^9 \frac{1}{\sqrt{u}} \frac{1}{3} du = \frac{1}{3} [2\sqrt{u}]_4^9 = \frac{2}{3}(\sqrt{9}-\sqrt{4}) = \frac{2}{3}.$$

(37) When $c_n = \frac{1}{\sqrt{n^2+n}} + \frac{1}{\sqrt{n^2+2n}} + \frac{1}{\sqrt{n^2+3n}} + \dots + \frac{1}{\sqrt{n^2+n^2}}$, find $\lim_{n \rightarrow \infty} c_n$.

(SOL)

$$\lim_{n \rightarrow \infty} c_n = \lim_{n \rightarrow \infty} \sum_{k=1}^n \frac{1}{\sqrt{n^2+kn}} = \lim_{n \rightarrow \infty} \sum_{k=1}^n \frac{1}{\sqrt{1+k/n}} \frac{1}{n} = \int_0^1 \frac{1}{\sqrt{1+x}} dx = 2\sqrt{1+x} \Big|_0^1 = 2\sqrt{2}-2.$$

(38) Find the region of the area enclosed by $x^3-3x^2+2x-5=0$ and $x^3-4x^2+3x+1=0$.

(SOL) From $x^3-3x^2+2x-5 = x^3-4x^2+3x+1$, it follows that $x^2-x-6 = (x-3)(x+2) = 0$.

Hence two graphs meet at $x=3, -2$. Hence, the area is given by

$$\begin{aligned} \int_{-2}^3 (-x^2+x+6) dx &= \left[-\frac{1}{3}x^3 + \frac{1}{2}x^2 + 6x \right]_{-2}^3 = \left(-9 + \frac{9}{2} + 18 \right) - \left(\frac{8}{3} + 2 - 12 \right) \\ &= 19 + \frac{9}{2} - \frac{8}{3} = \frac{114+27-16}{6} = \frac{125}{6}. \end{aligned}$$

(39) When $f(x)$ satisfies $f(x) = \frac{5}{3}x^3 - 6x^2 + 9x - \int_0^x f'(2t-3)dt$, find $f'(3)$.

(SOL) Taking derivative, it follows that $f'(x) = 5x^2 - 12x + 9 - f'(2x-3)$. Putting $x=3$, it follows that $f'(3) = 45 - 36 + 9 - f'(3)$, which is $2f'(3) = 18$. Hence, $f'(3) = 9$.

(40) When $\int_1^x tf(t)dt = \frac{4}{3}x^3 - 2x^2 + ax + b$ and $f(2)=5$ for some continuous function $f(x)$,

find ab .

(SOL) Putting $x=1$, it follows that $a+b = \frac{2}{3}$. Taking derivative, it follows that

$xf(x) = 4x^2 - 4x + a$. Putting $x=2$, it follows that $2f(2) = 8+a$. Since $f(2)=5$, it follows that $a=2$. Hence, $b = -\frac{4}{3}$ and $ab = -\frac{8}{3}$.

2025 IUT Admission Test(SOCIE)
Physics Examination(A TYPE)

<Multiple choice Types> There is only one correct answer per each question. Mark your answer choice on the OMR answer sheet.

- For each correct answer, you will get the points indicated next to each question number.
- No penalty point is applied to an incorrect answer.

1. [2 point]

As shown in the figure, the forces acting on the object are the force F , gravity mg , the normal force N , and the static friction force f_s . These forces can be divided into components in the x - and y -directions.

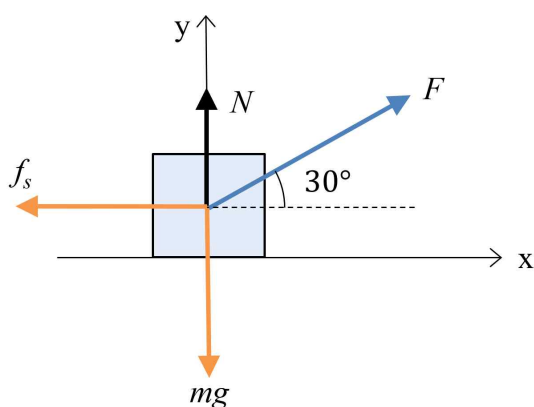
• y -direction: $N + F\sin 30^\circ - mg = 0$ ($F = 10$ N).

Therefore,

$$N = mg - (10 \text{ N}) \sin 30^\circ = (2 \text{ kg})(10 \text{ m/s}^2) - 5 \text{ N} = 15 \text{ N}.$$

• x -direction: $F \cos 30^\circ - f_s = 0$ ($f_s = \mu N$).

Therefore, $\mu = \frac{F \cos 30^\circ}{N} = \frac{(10 \text{ N})(\sqrt{3}/2)}{15 \text{ N}} = \frac{\sqrt{3}}{3}$.



Answer) ④ $\frac{\sqrt{3}}{3}$

2. [1 point]

When a train passes through a length L with a constant acceleration $a = 20 \text{ m/s}^2$, the initial and final velocities are $v_1 = 40 \text{ m/s}$ and $v_2 = 80 \text{ m/s}$, respectively. So from $2aL = v_2^2 - v_1^2$, we get

$$L = \frac{v_2^2 - v_1^2}{2a} = \frac{(80 \text{ m/s})^2 - (40 \text{ m/s})^2}{2(20 \text{ m/s}^2)} = 120 \text{ m}.$$

Answer) ② 120 m

3. [1 point]

The horizontal motion is of uniform velocity, and the vertical motion is of uniform acceleration. The time to reach the ground is the same as the free fall time,

from $h = \frac{1}{2}gt^2$, we get

$$t = \sqrt{\frac{2h}{g}} = \sqrt{\frac{2(500 \text{ m})}{10 \text{ m/s}^2}} = 10 \text{ s}.$$

Therefore, the

distance moved in the horizontal direction is $x = vt = 1000 \text{ m}$.

Answer) ① 1000 m

4. [2 points]

The gravitational force acting between the two twin stars acts as the centripetal force of the circular motion. Since the radius of the circular motion of the star is $\frac{r}{2}$, $G\frac{m^2}{r^2} = \frac{mv^2}{r/2}$. Therefore, since

$$v = \sqrt{\frac{Gm}{2r}}, \text{ the orbital period } T = \frac{\pi r}{v} = \pi \sqrt{\frac{2r^3}{Gm}}.$$

Answer) ③ $\pi \sqrt{\frac{2r^3}{Gm}}$

5. [2 point]

When the object falls to a position where the string is vertical, the object's velocity v_1 satisfies $mgl = \frac{1}{2}mv_1^2$ by the law of conservation of mechanical energy.

Therefore, $v_1 = \sqrt{2gl}$. Since the two objects collide elastically, momentum and kinetic energy are conserved before and after the collision. If the velocities of the two objects after the collision are v_2 and V_2 , $mv_1 = mv_2 + 2mV_2$ is established by conservation of momentum.

Therefore, $v_2 = v_1 - 2V_2$. -----(1)

Also, $\frac{1}{2}mv_1^2 = \frac{1}{2}mv_2^2 + \frac{1}{2}2mV_2^2$ is established by conservation of energy.

Therefore, $v_1^2 = v_2^2 + 2V_2^2$. -----(2)

Substituting equation (1) into equation (2), we get

$$V_2 = \frac{2}{3}v_1 = \frac{2}{3}\sqrt{2gl}.$$

Answer) ⑤ $\frac{2}{3}\sqrt{2gl}$

6. [1 points]

Impulse $J = \int F dt$, so impulse applied to the object for 4 seconds $J = (10 \text{ N})(4 \text{ s}) = 40 \text{ N}\cdot\text{s}$. Impulse is equal to the change in momentum, so $J = \Delta p = mv$.

Therefore $v = \frac{J}{m} = \frac{40 \text{ N}\cdot\text{s}}{5 \text{ kg}} = 8 \text{ m/s}$.

Answer) ③ 8 m/s

7. [2 points]

While an object is vibrating, mechanical energy is conserved. That is,

$$E = \frac{1}{2}kA^2 = \frac{1}{2}kx^2 + \frac{1}{2}mv^2 = \frac{1}{2}k\left(\frac{1}{2}A\right)^2 + \frac{1}{2}mv^2.$$

Therefore, the kinetic energy is $\frac{1}{2}mv^2 = \frac{3}{4} \times \frac{1}{2}kA^2$.

Therefore, the ratio of the potential energy to the

kinetic energy at point $\frac{A}{2}$ is

$$U : K = \frac{1}{2}k\left(\frac{A}{2}\right)^2 : \frac{3}{4} \times \frac{1}{2}kA^2 = 1 : 3.$$

Answer) ④ 1 : 3

8. [1 point]

The amount of heat required to change the temperature of a substance of mass m and specific heat c by ΔT is $Q = mc\Delta T$. Since the heat gained by the cold water is equal to the heat lost by the hot water, $Q = m_c c \Delta T_c = m_h c \Delta T_h$. Therefore, $m_c(30^\circ\text{C} - 10^\circ\text{C}) = m_h(60^\circ\text{C} - 30^\circ\text{C})$. Therefore, since $2m_c = 3m_h$ and $m_c = 900 \text{ g}$, then $m_h = 600 \text{ g}$.

Answer) ② 600 g

9. [2 points]

From the ideal gas equation $PV = nRT$, in the isometric process $A \rightarrow B$, the pressure and temperature are proportional, so the temperature at point B is $4T_0$. In the isobaric process $B \rightarrow C$, the volume and temperature are proportional, so the temperature at point C is $8T_0$. In the isothermal process $C \rightarrow D$, the temperature is constant and the volume is inversely proportional to the pressure, so the temperature at point D is $8T_0$ and the volume is $8V_0$.

Answer) ① $8T_0, 8V_0$

10. [1 points]

Objects A, B, and C move together due to a 12 N force acting on A. The acceleration a is obtained through the equation of motion $F = ma$ as follows.

$$a = \frac{F}{m} = \frac{12 \text{ N}}{(1 + 2 + 3) \text{ kg}} = 2 \text{ m/s}^2.$$

Therefore, the magnitude of the net force acting on B is $F_B = (2 \text{ kg})(2 \text{ m/s}^2) = 4 \text{ N}$.

Answer) ⑤ 4 N

11. [1 point]

The electric fields from the two charges must be equal in magnitude and opposite in direction, as follows: $E_1 = E_2$

Thus, we obtain:
$$\frac{4}{x^2} = \frac{1}{(6-x)^2}$$

Rearrange and factor the above equation, we obtain
$$(3x-12)(x-12) = 0$$

To ensure the vector summation of the electric fields results in zero, x must be located in the region $0 < x < 6$.

Therefore, x is found to be 4 m.

Answer) ④ 4 m

12. [2 point]

The electric potential V due to a single point charge q at a distance a is given by:

$$V_{single} = \frac{1}{4\pi\epsilon_0} \frac{q}{a}$$

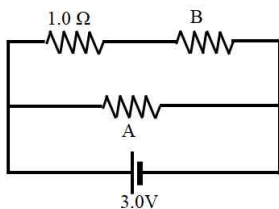
Electric potential is a scalar quantity, so the total potential at the center is the sum of the potentials from all six charges. Since all charges are identical and at the same distance:

$$V_{total} = 6 V_{single} = \frac{6}{4\pi\epsilon_0} \frac{q}{a}$$

Answer) ④ $\frac{6}{4\pi\epsilon_0} \frac{q}{a}$

13. [2 point]

The equivalent circuit is presented as follows:



From the ratio of voltage, the voltage difference across resistor B is 2V.

By Kirchhoff's loop rule, the voltage difference across the 1.0 ohm resistor is 1 V. So the current through the 1.0 ohm resistor is 1 A. Therefore, we obtain

$$R_B = \frac{2V}{1A} = 2\text{ ohm.}$$

Answer) ② 2 ohm

14. [1 point]

Given that the lens formula $\frac{1}{o} + \frac{1}{i} = \frac{1}{f}$.

By substitute the given values, we obtain

$$\frac{1}{60\text{ cm}} + \frac{1}{i} = \frac{1}{10\text{ cm}}$$

solving the above equation, $i = 12\text{ cm}$.

Answer) ⑤ 12 cm

15. [2 points]

Total capacitance:

$$\frac{1}{C_{total}} = \frac{1}{4\mu\text{F}} + \frac{1}{4\mu\text{F}} = \frac{1}{2\mu\text{F}} \therefore C_{total} = 2\mu\text{F}$$

The energy stored in a capacitor is given by:

$$U = \frac{1}{2} C_{total} V^2$$

Thus, we obtain $\frac{1}{2}(2\mu\text{F})(3\text{V})^2 = 9\mu\text{J}$.

Answer) ② 9 μJ

16. [1 point]

By Ohm's Law, where the voltage is 6V and the total resistance is 6Ω, so the current I is 1A.

The power consumed by a resistor is given by

$$P = I^2 R.$$

Thus, we obtain $(1\text{A})^2(2\text{ohm}) = 2\text{W}$.

Answer) ② 2 W

17. [1 points]

Assuming an ideal transformer with no power loss, the power P remains constant.

Thus, $14,000\text{V} \times 10\text{A} = 140,000\text{V} \times I$.

Solving for the new current I ,

$$I = (14,000 \times 10) / 140,000 = 1 \text{ A.}$$

Thus, the resulting current after the voltage step-up is 1 A.

Answer) ① 1 A

18. [1 points]

The magnetic field B around a long, straight wire carrying current I is given by Ampère's law:

$$B = \frac{\mu_0 I}{2\pi d}$$

Therefore, we obtain each magnetic field as below:

(1) At distance d , $B_d = \frac{\mu_0 I}{2\pi d}$

(2) At distance $2d$, $B_{2d} = \frac{\mu_0 I}{4\pi d}$

Thus, the ratio of the magnetic field strengths is:

$$\frac{B_d}{B_{2d}} = 2$$

Answer) ③ 2

19. [2 point]

Young's double-slit experiment gives the fringe

spacing Δy as: $\Delta y = \frac{\lambda L}{d}$

Rearrange the formula to solve for λ : $\lambda = \frac{(\Delta y)(d)}{L}$

By substituting the given conditions, we obtain

$$\frac{(2.5 \times 10^{-2})(10^{-4})}{5} = 5 \times 10^{-7} \text{ m.}$$

Therefore, the wavelength of the laser light is 500 nm.

Answer) ⑤ 500 nm

20. [2 points]

The de Broglie wavelength λ of a particle is given

$$\text{by: } \lambda = \frac{h}{p} = \frac{h}{mv}$$

Rearrange to solve for the speed v : $v = \frac{h}{m\lambda}$

By substituting the given conditions, we obtain

$$v = \frac{6.6 \times 10^{-34}}{(2 \times 10^{-27})(6.6 \times 10^{-10})} = 500 \text{ m/s.}$$

Therefore, the speed of the object is 500 m/s.

Answer) ④ 500 m/s

2025 IUT Admission Test(SOCIE)
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Answers

1. ④
2. ②
3. ①
4. ③
5. ⑤
6. ③
7. ④
8. ②
9. ①
10. ⑤
11. ④
12. ④
13. ②
14. ⑤
15. ②
16. ②
17. ①
18. ③
19. ⑤
20. ④