

Math Examination(TYPE A)

< Multiple choice Types > There is only one correct answer for each question. Mark your choice on the OMR answer sheet.

- The points for each question are listed next to the question number.
- You can use the right side of each page for your memo.

1. [3 points]

When $(2 + \sqrt{3})^2 = a + b\sqrt{3}$ for integers a and b , find $a + b$.

- ① 7 ② 9 ③ 11 ④ 13 ⑤ 15

2. [3 points]

Simplify $\sqrt{2} \times \sqrt[4]{64}$.

- ① $2\sqrt{2}$ ② 4 ③ $4\sqrt{2}$ ④ 8 ⑤ $8\sqrt{2}$

3. [3 points]

When $\alpha + \frac{1}{\alpha} = 3$, find $\alpha^2 + \frac{1}{\alpha^2}$.

- ① 1 ② 3 ③ 5 ④ 7 ⑤ 9

4. [3 points]

When α and β are the solutions of $x^2 - 2x + 4 = 0$, find $\frac{1}{\alpha} + \frac{1}{\beta}$.

- ① $\frac{1}{2}$ ② 1 ③ $\frac{3}{2}$ ④ 2 ⑤ $\frac{5}{2}$

5. [3 points]

When α, β, γ are the solutions of $x^3 - 4x^2 + 3x - 4 = 0$, find $\alpha^2 + \beta^2 + \gamma^2$.

- ① 6 ② 7 ③ 8 ④ 9 ⑤ 10

6. [3 points]

When $(x, y) = (\alpha, \beta)$ is the solution of

$$\begin{cases} 2x + y = 4 \\ x - 3y = 3 \end{cases}$$

find $\alpha + \beta$.

- ① 2 ② $\frac{13}{7}$ ③ $\frac{7}{4}$ ④ $\frac{5}{3}$ ⑤ $\frac{8}{5}$

7. [3 points]

Compute $\sum_{k=1}^{100} k$.

- ① 5040 ② 5045 ③ 5050 ④ 5055 ⑤ 5060

8. [3 points]

When $\log_4 a = \frac{3}{2}$, find a .

- ① 6 ② 8 ③ 10 ④ 12 ⑤ 14

9. [3 points]

Find $\log_2 8 + \log_2 \frac{1}{16}$.

- ① -1 ② $-\frac{1}{2}$ ③ 0 ④ $\frac{1}{2}$ ⑤ 1

10. [3 points]

Find the sum of all real solutions of

$$6 \cdot 3^x + 2 \cdot 3^{-x} = 7.$$

- ① -1 ② $-\frac{1}{2}$ ③ 0 ④ $\frac{1}{2}$ ⑤ 1

11. [3 points]

When $2^a = \sqrt{3}$ and $3^b = 8$, find ab .

- ① $\frac{1}{3}$ ② $\frac{2}{3}$ ③ $\frac{1}{2}$ ④ 1 ⑤ $\frac{3}{2}$

12. [3 points]

When $(2+i)^3 = a+bi$ for real numbers a and b , find $a+b$.

- ① 11 ② 13 ③ 15 ④ 17 ⑤ 19

13. [3 points]

When $\frac{2-i}{1+i} = a+bi$ for real numbers a and b , find $a+b$.

- ① -4 ② -3 ③ -2 ④ -1 ⑤ 0

14. [3 points]

Find $\sin \frac{5\pi}{12}$.

- ① $\frac{1+\sqrt{2}}{4}$ ② $\frac{1+\sqrt{3}}{4}$ ③ $\frac{1+\sqrt{6}}{4}$
④ $\frac{\sqrt{2}+\sqrt{3}}{4}$ ⑤ $\frac{\sqrt{2}+\sqrt{6}}{4}$

15. [3 points]

Find the sum of all solutions of $\cos 2x = \sin x$, ($0 \leq x < 2\pi$).

- ① π ② $\frac{3\pi}{2}$ ③ 2π ④ $\frac{5\pi}{2}$ ⑤ 3π

16. [3 points]

When $\sin 2\theta = \sin \theta$ for $0 < \theta < \pi$, find $\tan \theta$,

where $\tan \theta = \frac{\sin \theta}{\cos \theta}$.

- ① $\frac{\sqrt{3}}{3}$ ② $\frac{2\sqrt{3}}{3}$ ③ $\sqrt{3}$
④ $\frac{4\sqrt{3}}{3}$ ⑤ $\frac{5\sqrt{3}}{3}$

17. [3 points]

When $A = \begin{pmatrix} 1 & 1 \\ -3 & -5 \end{pmatrix}$ and $A^{-1} = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$,

find $a+b+c+d$.

- ① -3 ② -1 ③ 0 ④ 1 ⑤ 3

18. [3 points]

When $A = \begin{pmatrix} -1 & 2 \\ 1 & 3 \end{pmatrix}$, $B = \begin{pmatrix} 1 & 2 \\ 1 & 1 \end{pmatrix}$, and

$AB^{-1} = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$, find $a+b+c+d$.

- ① 0 ② 2 ③ 4 ④ 6 ⑤ 8

19. [3 points]

Find $\lim_{x \rightarrow \infty} \frac{x}{\sqrt{4x^2 + 3x + 2}}$.

- ① $\frac{1}{\sqrt{6}}$ ② $\frac{1}{\sqrt{5}}$ ③ $\frac{1}{2}$ ④ $\frac{1}{\sqrt{3}}$ ⑤ $\frac{1}{\sqrt{2}}$

20. [3 points]

Find $\lim_{x \rightarrow 1} \frac{x-1}{2x^2 - x - 1}$.

- ① $\frac{1}{2}$ ② $\frac{1}{3}$ ③ $\frac{1}{4}$ ④ $\frac{1}{5}$ ⑤ $\frac{1}{6}$

21. [4 points]

Find $\lim_{x \rightarrow 1} \frac{\sin(x-1)}{x^2 + 3x - 4}$.

- ① 1 ② $\frac{1}{2}$ ③ $\frac{1}{3}$ ④ $\frac{1}{4}$ ⑤ $\frac{1}{5}$

22. [4 points]

When $\lim_{x \rightarrow \infty} \frac{f(x)}{3x+2} = 2$ and $\lim_{x \rightarrow \infty} \frac{2x+1}{g(x)} = 3$,

find $\lim_{x \rightarrow \infty} \frac{f(x)}{g(x)}$.

- ① 5 ② 6 ③ 7 ④ 8 ⑤ 9

23. [4 points]

When $f(x) = (2x^2 + x + 1)^3$, find $f'(-1)$.

- ① -36 ② -34 ③ -32 ④ -30 ⑤ -28

24. [4 points]

When $f(x) = \frac{3}{\sqrt{x+1}}$, find $f'(4)$.

- ① $-\frac{1}{2}$ ② $-\frac{1}{3}$ ③ $-\frac{1}{4}$ ④ $-\frac{1}{6}$ ⑤ $-\frac{1}{12}$

25. [4 points]

When $y = ax + b$, for some constants a, b , is the tangent line to $y = 3x^2 + x$ at $x = -1$, find $a + b$.

- ① -2 ② -4 ③ -6 ④ -8 ⑤ -10

26. [4 points]

When M and m are the maximum and minimum values of $f(x) = \frac{2}{3}x^3 - x^2 + 1$, ($0 \leq x \leq 2$), find $M + m$.

- ① $\frac{5}{3}$ ② 2 ③ $\frac{7}{3}$ ④ $\frac{8}{3}$ ⑤ 3

27. [4 points]

Compute $\int_0^1 (2x^2 + x) dx$.

- ① $\frac{2}{3}$ ② $\frac{5}{6}$ ③ 1 ④ $\frac{7}{6}$ ⑤ $\frac{4}{3}$

28. [4 points]

Compute $\int_0^1 x(x^2 + 1)^3 dx$.

- ① $\frac{13}{8}$ ② $\frac{7}{4}$ ③ $\frac{15}{8}$ ④ 2 ⑤ $\frac{17}{8}$

29. [4 points]

Find the area of the region enclosed by $y = x^3 - 2x^2 - 4x + 2$ and $y = x^3 - 3x^2 - 2x + 2$.

- ① $\frac{2}{3}$ ② $\frac{4}{3}$ ③ 2 ④ $\frac{8}{3}$ ⑤ $\frac{10}{3}$

30. [4 points]

When $\int_1^x tf(t^2) dt = x^4 + ax^2$ for a differentiable function $f(x)$ and a constant a , find $\int_0^3 f(x) dx$.

- ① 6 ② 9 ③ 12 ④ 15 ⑤ 18

Math Examination(TYPE B)

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1. [3 points]

When α, β, γ are the solutions of $x^3 - 4x^2 + 3x - 4 = 0$, find $\alpha^2 + \beta^2 + \gamma^2$.

- ① 6 ② 7 ③ 8 ④ 9 ⑤ 10

2. [3 points]

When $(2 + \sqrt{3})^2 = a + b\sqrt{3}$ for integers a and b , find $a + b$.

- ① 7 ② 9 ③ 11 ④ 13 ⑤ 15

3. [3 points]

Simplify $\sqrt{2} \times \sqrt[4]{64}$.

- ① $2\sqrt{2}$ ② 4 ③ $4\sqrt{2}$ ④ 8 ⑤ $8\sqrt{2}$

4. [3 points]

When $\alpha + \frac{1}{\alpha} = 3$, find $\alpha^2 + \frac{1}{\alpha^2}$.

- ① 1 ② 3 ③ 5 ④ 7 ⑤ 9

5. [3 points]

When α and β are the solutions of $x^2 - 2x + 4 = 0$, find $\frac{1}{\alpha} + \frac{1}{\beta}$.

- ① $\frac{1}{2}$ ② 1 ③ $\frac{3}{2}$ ④ 2 ⑤ $\frac{5}{2}$

6. [3 points]

Find the sum of all real solutions of $6 \cdot 3^x + 2 \cdot 3^{-x} = 7$.

- ① -1 ② $-\frac{1}{2}$ ③ 0 ④ $\frac{1}{2}$ ⑤ 1

7. [3 points]

When $(x, y) = (\alpha, \beta)$ is the solution of

$$\begin{cases} 2x + y = 4 \\ x - 3y = 3 \end{cases}$$

find $\alpha + \beta$.

- ① 2 ② $\frac{13}{7}$ ③ $\frac{7}{4}$ ④ $\frac{5}{3}$ ⑤ $\frac{8}{5}$

8. [3 points]

Compute $\sum_{k=1}^{100} k$.

- ① 5040 ② 5045 ③ 5050 ④ 5055 ⑤ 5060

9. [3 points]

When $\log_4 a = \frac{3}{2}$, find a .

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10. [3 points]

Find $\log_2 8 + \log_2 \frac{1}{16}$.

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11. [3 points]

Find the sum of all solutions of $\cos 2x = \sin x$, ($0 \leq x < 2\pi$).

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12. [3 points]

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- ① $\frac{1}{3}$ ② $\frac{2}{3}$ ③ $\frac{1}{2}$ ④ 1 ⑤ $\frac{3}{2}$

13. [3 points]

When $(2+i)^3 = a+bi$ for real numbers a and b , find $a+b$.

- ① 11 ② 13 ③ 15 ④ 17 ⑤ 19

14. [3 points]

When $\frac{2-i}{1+i} = a+bi$ for real numbers a and b , find $a+b$.

- ① -4 ② -3 ③ -2 ④ -1 ⑤ 0

15. [3 points]

Find $\sin \frac{5\pi}{12}$.

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④ $\frac{\sqrt{2}+\sqrt{3}}{4}$ ⑤ $\frac{\sqrt{2}+\sqrt{6}}{4}$

16. [3 points]

Find $\lim_{x \rightarrow 1} \frac{x-1}{2x^2-x-1}$.

- ① $\frac{1}{2}$ ② $\frac{1}{3}$ ③ $\frac{1}{4}$ ④ $\frac{1}{5}$ ⑤ $\frac{1}{6}$

17. [3 points]

When $\sin 2\theta = \sin \theta$ for $0 < \theta < \pi$, find $\operatorname{tg} \theta$, where $\operatorname{tg} \theta = \frac{\sin \theta}{\cos \theta}$.

- ① $\frac{\sqrt{3}}{3}$ ② $\frac{2\sqrt{3}}{3}$ ③ $\sqrt{3}$
④ $\frac{4\sqrt{3}}{3}$ ⑤ $\frac{5\sqrt{3}}{3}$

18. [3 points]

When $A = \begin{pmatrix} 1 & 1 \\ -3 & -5 \end{pmatrix}$ and $A^{-1} = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$, find $a+b+c+d$.

- ① -3 ② -1 ③ 0 ④ 1 ⑤ 3

19. [3 points]

When $A = \begin{pmatrix} -1 & 2 \\ 1 & 3 \end{pmatrix}$, $B = \begin{pmatrix} 1 & 2 \\ 1 & 1 \end{pmatrix}$, and $AB^{-1} = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$, find $a+b+c+d$.

- ① 0 ② 2 ③ 4 ④ 6 ⑤ 8

20. [3 points]

Find $\lim_{x \rightarrow \infty} \frac{x}{\sqrt{4x^2+3x+2}}$.

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21. [4 points]

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- ① -2 ② -4 ③ -6 ④ -8 ⑤ -10

22. [4 points]

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- ① 1 ② $\frac{1}{2}$ ③ $\frac{1}{3}$ ④ $\frac{1}{4}$ ⑤ $\frac{1}{5}$

23. [4 points]

When $\lim_{x \rightarrow \infty} \frac{f(x)}{3x+2} = 2$ and $\lim_{x \rightarrow \infty} \frac{2x+1}{g(x)} = 3$,

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24. [4 points]

When $f(x) = (2x^2 + x + 1)^3$, find $f'(-1)$.

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- ① $-\frac{1}{2}$ ② $-\frac{1}{3}$ ③ $-\frac{1}{4}$ ④ $-\frac{1}{6}$ ⑤ $-\frac{1}{12}$

26. [4 points]

When $\int_1^x tf(t^2) dt = x^4 + ax^2$ for a differentiable function $f(x)$ and a constant a , find $\int_0^3 f(x) dx$.

- ① 6 ② 9 ③ 12 ④ 15 ⑤ 18

27. [4 points]

When M and m are the maximum and minimum values of $f(x) = \frac{2}{3}x^3 - x^2 + 1$, ($0 \leq x \leq 2$), find $M + m$.

- ① $\frac{5}{3}$ ② 2 ③ $\frac{7}{3}$ ④ $\frac{8}{3}$ ⑤ 3

28. [4 points]

Compute $\int_0^1 (2x^2 + x) dx$.

- ① $\frac{2}{3}$ ② $\frac{5}{6}$ ③ 1 ④ $\frac{7}{6}$ ⑤ $\frac{4}{3}$

29. [4 points]

Compute $\int_0^1 x(x^2 + 1)^3 dx$.

- ① $\frac{13}{8}$ ② $\frac{7}{4}$ ③ $\frac{15}{8}$ ④ 2 ⑤ $\frac{17}{8}$

30. [4 points]

Find the area of the region enclosed by $y = x^3 - 2x^2 - 4x + 2$ and $y = x^3 - 3x^2 - 2x + 2$.

- ① $\frac{2}{3}$ ② $\frac{4}{3}$ ③ 2 ④ $\frac{8}{3}$ ⑤ $\frac{10}{3}$

Math Examination(TYPE C)

< Multiple choice Types > There is only one correct answer for each question. Mark your choice on the OMR answer sheet.

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1. [3 points]

When α and β are the solutions of

$$x^2 - 2x + 4 = 0, \text{ find } \frac{1}{\alpha} + \frac{1}{\beta}.$$

- ① $\frac{1}{2}$ ② 1 ③ $\frac{3}{2}$ ④ 2 ⑤ $\frac{5}{2}$

2. [3 points]

When α, β, γ are the solutions of

$$x^3 - 4x^2 + 3x - 4 = 0, \text{ find } \alpha^2 + \beta^2 + \gamma^2.$$

- ① 6 ② 7 ③ 8 ④ 9 ⑤ 10

3. [3 points]

When $(2 + \sqrt{3})^2 = a + b\sqrt{3}$ for integers a and b , find $a + b$.

- ① 7 ② 9 ③ 11 ④ 13 ⑤ 15

4. [3 points]

Simplify $\sqrt{2} \times \sqrt[4]{64}$.

- ① $2\sqrt{2}$ ② 4 ③ $4\sqrt{2}$ ④ 8 ⑤ $8\sqrt{2}$

5. [3 points]

When $\alpha + \frac{1}{\alpha} = 3$, find $\alpha^2 + \frac{1}{\alpha^2}$.

- ① 1 ② 3 ③ 5 ④ 7 ⑤ 9

6. [3 points]

Find $\log_2 8 + \log_2 \frac{1}{16}$.

- ① -1 ② $-\frac{1}{2}$ ③ 0 ④ $\frac{1}{2}$ ⑤ 1

7. [3 points]

Find the sum of all real solutions of

$$6 \cdot 3^x + 2 \cdot 3^{-x} = 7.$$

- ① -1 ② $-\frac{1}{2}$ ③ 0 ④ $\frac{1}{2}$ ⑤ 1

8. [3 points]

When $(x, y) = (\alpha, \beta)$ is the solution of

$$\begin{cases} 2x + y = 4 \\ x - 3y = 3, \end{cases}$$

find $\alpha + \beta$.

- ① 2 ② $\frac{13}{7}$ ③ $\frac{7}{4}$ ④ $\frac{5}{3}$ ⑤ $\frac{8}{5}$

9. [3 points]

Compute $\sum_{k=1}^{100} k$.

- ① 5040 ② 5045 ③ 5050 ④ 5055 ⑤ 5060

10. [3 points]

When $\log_4 a = \frac{3}{2}$, find a .

- ① 6 ② 8 ③ 10 ④ 12 ⑤ 14

11. [3 points]

Find $\sin \frac{5\pi}{12}$.

- ① $\frac{1+\sqrt{2}}{4}$ ② $\frac{1+\sqrt{3}}{4}$ ③ $\frac{1+\sqrt{6}}{4}$
④ $\frac{\sqrt{2}+\sqrt{3}}{4}$ ⑤ $\frac{\sqrt{2}+\sqrt{6}}{4}$

12. [3 points]

Find the sum of all solutions of $\cos 2x = \sin x$, ($0 \leq x < 2\pi$).

- ① π ② $\frac{3\pi}{2}$ ③ 2π ④ $\frac{5\pi}{2}$ ⑤ 3π

13. [3 points]

When $2^a = \sqrt{3}$ and $3^b = 8$, find ab .

- ① $\frac{1}{3}$ ② $\frac{2}{3}$ ③ $\frac{1}{2}$ ④ 1 ⑤ $\frac{3}{2}$

14. [3 points]

When $(2+i)^3 = a+bi$ for real numbers a and b , find $a+b$.

- ① 11 ② 13 ③ 15 ④ 17 ⑤ 19

15. [3 points]

When $\frac{2-i}{1+i} = a+bi$ for real numbers a and b , find $a+b$.

- ① -4 ② -3 ③ -2 ④ -1 ⑤ 0

16. [3 points]

Find $\lim_{x \rightarrow \infty} \frac{x}{\sqrt{4x^2 + 3x + 2}}$.

- ① $\frac{1}{\sqrt{6}}$ ② $\frac{1}{\sqrt{5}}$ ③ $\frac{1}{2}$ ④ $\frac{1}{\sqrt{3}}$ ⑤ $\frac{1}{\sqrt{2}}$

17. [3 points]

Find $\lim_{x \rightarrow 1} \frac{x-1}{2x^2 - x - 1}$.

- ① $\frac{1}{2}$ ② $\frac{1}{3}$ ③ $\frac{1}{4}$ ④ $\frac{1}{5}$ ⑤ $\frac{1}{6}$

18. [3 points]

When $\sin 2\theta = \sin \theta$ for $0 < \theta < \pi$, find $\operatorname{tg} \theta$, where $\operatorname{tg} \theta = \frac{\sin \theta}{\cos \theta}$.

- ① $\frac{\sqrt{3}}{3}$ ② $\frac{2\sqrt{3}}{3}$ ③ $\sqrt{3}$
④ $\frac{4\sqrt{3}}{3}$ ⑤ $\frac{5\sqrt{3}}{3}$

19. [3 points]

When $A = \begin{pmatrix} 1 & 1 \\ -3 & -5 \end{pmatrix}$ and $A^{-1} = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$, find $a+b+c+d$.

- ① -3 ② -1 ③ 0 ④ 1 ⑤ 3

20. [3 points]

When $A = \begin{pmatrix} -1 & 2 \\ 1 & 3 \end{pmatrix}$, $B = \begin{pmatrix} 1 & 2 \\ 1 & 1 \end{pmatrix}$, and $AB^{-1} = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$, find $a+b+c+d$.

- ① 0 ② 2 ③ 4 ④ 6 ⑤ 8

21. [4 points]

When $f(x) = \frac{3}{\sqrt{x+1}}$, find $f'(4)$.

- ① $-\frac{1}{2}$ ② $-\frac{1}{3}$ ③ $-\frac{1}{4}$ ④ $-\frac{1}{6}$ ⑤ $-\frac{1}{12}$

22. [4 points]

When $y = ax + b$, for some constants a, b , is the tangent line to $y = 3x^2 + x$ at $x = -1$, find $a + b$.

- ① -2 ② -4 ③ -6 ④ -8 ⑤ -10

23. [4 points]

Find $\lim_{x \rightarrow 1} \frac{\sin(x-1)}{x^2 + 3x - 4}$.

- ① 1 ② $\frac{1}{2}$ ③ $\frac{1}{3}$ ④ $\frac{1}{4}$ ⑤ $\frac{1}{5}$

24. [4 points]

When $\lim_{x \rightarrow \infty} \frac{f(x)}{3x+2} = 2$ and $\lim_{x \rightarrow \infty} \frac{2x+1}{g(x)} = 3$,

find $\lim_{x \rightarrow \infty} \frac{f(x)}{g(x)}$.

- ① 5 ② 6 ③ 7 ④ 8 ⑤ 9

25. [4 points]

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26. [4 points]

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$$y = x^3 - 2x^2 - 4x + 2 \quad \text{and} \quad y = x^3 - 3x^2 - 2x + 2.$$

- ① $\frac{2}{3}$ ② $\frac{4}{3}$ ③ 2 ④ $\frac{8}{3}$ ⑤ $\frac{10}{3}$

27. [4 points]

When $\int_1^x tf(t^2) dt = x^4 + ax^2$ for a differentiable function $f(x)$ and a constant a , find $\int_0^3 f(x) dx$.

- ① 6 ② 9 ③ 12 ④ 15 ⑤ 18

28. [4 points]

When M and m are the maximum and minimum values of $f(x) = \frac{2}{3}x^3 - x^2 + 1$, ($0 \leq x \leq 2$), find $M + m$.

- ① $\frac{5}{3}$ ② 2 ③ $\frac{7}{3}$ ④ $\frac{8}{3}$ ⑤ 3

29. [4 points]

Compute $\int_0^1 (2x^2 + x) dx$.

- ① $\frac{2}{3}$ ② $\frac{5}{6}$ ③ 1 ④ $\frac{7}{6}$ ⑤ $\frac{4}{3}$

30. [4 points]

Compute $\int_0^1 x(x^2 + 1)^3 dx$.

- ① $\frac{13}{8}$ ② $\frac{7}{4}$ ③ $\frac{15}{8}$ ④ 2 ⑤ $\frac{17}{8}$

Math Examination(TYPE D)

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1. [3 points]

When $\alpha + \frac{1}{\alpha} = 3$, find $\alpha^2 + \frac{1}{\alpha^2}$.

- ① 1 ② 3 ③ 5 ④ 7 ⑤ 9

2. [3 points]

When α and β are the solutions of $x^2 - 2x + 4 = 0$, find $\frac{1}{\alpha} + \frac{1}{\beta}$.

- ① $\frac{1}{2}$ ② 1 ③ $\frac{3}{2}$ ④ 2 ⑤ $\frac{5}{2}$

3. [3 points]

When α, β, γ are the solutions of $x^3 - 4x^2 + 3x - 4 = 0$, find $\alpha^2 + \beta^2 + \gamma^2$.

- ① 6 ② 7 ③ 8 ④ 9 ⑤ 10

4. [3 points]

When $(2 + \sqrt{3})^2 = a + b\sqrt{3}$ for integers a and b , find $a + b$.

- ① 7 ② 9 ③ 11 ④ 13 ⑤ 15

5. [3 points]

Simplify $\sqrt{2} \times \sqrt[4]{64}$.

- ① $2\sqrt{2}$ ② 4 ③ $4\sqrt{2}$ ④ 8 ⑤ $8\sqrt{2}$

6. [3 points]

When $\log_4 a = \frac{3}{2}$, find a .

- ① 6 ② 8 ③ 10 ④ 12 ⑤ 14

7. [3 points]

Find $\log_2 8 + \log_2 \frac{1}{16}$.

- ① -1 ② $-\frac{1}{2}$ ③ 0 ④ $\frac{1}{2}$ ⑤ 1

8. [3 points]

Find the sum of all real solutions of $6 \cdot 3^x + 2 \cdot 3^{-x} = 7$.

- ① -1 ② $-\frac{1}{2}$ ③ 0 ④ $\frac{1}{2}$ ⑤ 1

9. [3 points]

When $(x, y) = (\alpha, \beta)$ is the solution of

$$\begin{cases} 2x + y = 4 \\ x - 3y = 3 \end{cases}$$

find $\alpha + \beta$.

- ① 2 ② $\frac{13}{7}$ ③ $\frac{7}{4}$ ④ $\frac{5}{3}$ ⑤ $\frac{8}{5}$

10. [3 points]

Compute $\sum_{k=1}^{100} k$.

- ① 5040 ② 5045 ③ 5050 ④ 5055 ⑤ 5060

11. [3 points]

When $\frac{2-i}{1+i} = a+bi$ for real numbers a and b , find $a+b$.

- ① -4 ② -3 ③ -2 ④ -1 ⑤ 0

12. [3 points]

Find $\sin \frac{5\pi}{12}$.

- ① $\frac{1+\sqrt{2}}{4}$ ② $\frac{1+\sqrt{3}}{4}$ ③ $\frac{1+\sqrt{6}}{4}$
④ $\frac{\sqrt{2}+\sqrt{3}}{4}$ ⑤ $\frac{\sqrt{2}+\sqrt{6}}{4}$

13. [3 points]

Find the sum of all solutions of $\cos 2x = \sin x$, ($0 \leq x < 2\pi$).

- ① π ② $\frac{3\pi}{2}$ ③ 2π ④ $\frac{5\pi}{2}$ ⑤ 3π

14. [3 points]

When $2^a = \sqrt{3}$ and $3^b = 8$, find ab .

- ① $\frac{1}{3}$ ② $\frac{2}{3}$ ③ $\frac{1}{2}$ ④ 1 ⑤ $\frac{3}{2}$

15. [3 points]

When $(2+i)^3 = a+bi$ for real numbers a and b , find $a+b$.

- ① 11 ② 13 ③ 15 ④ 17 ⑤ 19

16. [3 points]

When $A = \begin{pmatrix} -1 & 2 \\ 1 & 3 \end{pmatrix}$, $B = \begin{pmatrix} 1 & 2 \\ 1 & 1 \end{pmatrix}$, and $AB^{-1} = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$, find $a+b+c+d$.

- ① 0 ② 2 ③ 4 ④ 6 ⑤ 8

17. [3 points]

Find $\lim_{x \rightarrow \infty} \frac{x}{\sqrt{4x^2+3x+2}}$.

- ① $\frac{1}{\sqrt{6}}$ ② $\frac{1}{\sqrt{5}}$ ③ $\frac{1}{2}$ ④ $\frac{1}{\sqrt{3}}$ ⑤ $\frac{1}{\sqrt{2}}$

18. [3 points]

Find $\lim_{x \rightarrow 1} \frac{x-1}{2x^2-x-1}$.

- ① $\frac{1}{2}$ ② $\frac{1}{3}$ ③ $\frac{1}{4}$ ④ $\frac{1}{5}$ ⑤ $\frac{1}{6}$

19. [3 points]

When $\sin 2\theta = \sin \theta$ for $0 < \theta < \pi$, find $\tan \theta$, where $\tan \theta = \frac{\sin \theta}{\cos \theta}$.

- ① $\frac{\sqrt{3}}{3}$ ② $\frac{2\sqrt{3}}{3}$ ③ $\sqrt{3}$
④ $\frac{4\sqrt{3}}{3}$ ⑤ $\frac{5\sqrt{3}}{3}$

20. [3 points]

When $A = \begin{pmatrix} 1 & 1 \\ -3 & -5 \end{pmatrix}$ and $A^{-1} = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$, find $a+b+c+d$.

- ① -3 ② -1 ③ 0 ④ 1 ⑤ 3

21. [4 points]

When $f(x) = (2x^2 + x + 1)^3$, find $f'(-1)$.

- ① -36 ② -34 ③ -32 ④ -30 ⑤ -28

22. [4 points]

When $f(x) = \frac{3}{\sqrt{x+1}}$, find $f'(4)$.

- ① $-\frac{1}{2}$ ② $-\frac{1}{3}$ ③ $-\frac{1}{4}$ ④ $-\frac{1}{6}$ ⑤ $-\frac{1}{12}$

23. [4 points]

When $y = ax + b$, for some constants a, b , is the tangent line to $y = 3x^2 + x$ at $x = -1$, find $a + b$.

- ① -2 ② -4 ③ -6 ④ -8 ⑤ -10

24. [4 points]

Find $\lim_{x \rightarrow 1} \frac{\sin(x-1)}{x^2 + 3x - 4}$.

- ① 1 ② $\frac{1}{2}$ ③ $\frac{1}{3}$ ④ $\frac{1}{4}$ ⑤ $\frac{1}{5}$

25. [4 points]

When $\lim_{x \rightarrow \infty} \frac{f(x)}{3x+2} = 2$ and $\lim_{x \rightarrow \infty} \frac{2x+1}{g(x)} = 3$,

find $\lim_{x \rightarrow \infty} \frac{f(x)}{g(x)}$.

- ① 5 ② 6 ③ 7 ④ 8 ⑤ 9

26. [4 points]

Compute $\int_0^1 x(x^2 + 1)^3 dx$.

- ① $\frac{13}{8}$ ② $\frac{7}{4}$ ③ $\frac{15}{8}$ ④ 2 ⑤ $\frac{17}{8}$

27. [4 points]

Find the area of the region enclosed by

$y = x^3 - 2x^2 - 4x + 2$ and $y = x^3 - 3x^2 - 2x + 2$.

- ① $\frac{2}{3}$ ② $\frac{4}{3}$ ③ 2 ④ $\frac{8}{3}$ ⑤ $\frac{10}{3}$

28. [4 points]

When $\int_1^x tf(t^2) dt = x^4 + ax^2$ for a differentiable

function $f(x)$ and a constant a , find $\int_0^3 f(x) dx$.

- ① 6 ② 9 ③ 12 ④ 15 ⑤ 18

29. [4 points]

When M and m are the maximum and

minimum values of $f(x) = \frac{2}{3}x^3 - x^2 + 1$,

($0 \leq x \leq 2$), find $M + m$.

- ① $\frac{5}{3}$ ② 2 ③ $\frac{7}{3}$ ④ $\frac{8}{3}$ ⑤ 3

30. [4 points]

Compute $\int_0^1 (2x^2 + x) dx$.

- ① $\frac{2}{3}$ ② $\frac{5}{6}$ ③ 1 ④ $\frac{7}{6}$ ⑤ $\frac{4}{3}$

2025 IUT 3rd SBL Non-Scholarship Answer Sheets

[TypeA]

No.	1	2	3	4	5	6	7	8	9	10
Ans.	③	②	④	①	⑤	②	③	②	①	①
No.	11	12	13	14	15	16	17	18	19	20
Ans.	⑤	②	④	⑤	④	③	④	①	③	②
No.	21	22	23	24	25	26	27	28	29	30
Ans.	⑤	⑤	①	⑤	④	⑤	④	③	②	③

[TypeB]

No.	1	2	3	4	5	6	7	8	9	10
Ans.	⑤	③	②	④	①	①	②	③	②	①
No.	11	12	13	14	15	16	17	18	19	20
Ans.	④	⑤	②	④	⑤	②	③	④	①	③
No.	21	22	23	24	25	26	27	28	29	30
Ans.	④	⑤	⑤	①	⑤	③	⑤	④	③	②

[TypeC]

No.	1	2	3	4	5	6	7	8	9	10
Ans.	①	⑤	③	②	④	①	①	②	③	②
No.	11	12	13	14	15	16	17	18	19	20
Ans.	⑤	④	⑤	②	④	③	②	③	④	①
No.	21	22	23	24	25	26	27	28	29	30
Ans.	⑤	④	⑤	⑤	①	②	③	⑤	④	③

[TypeD]

No.	1	2	3	4	5	6	7	8	9	10
Ans.	④	①	⑤	③	②	②	①	①	②	③
No.	11	12	13	14	15	16	17	18	19	20
Ans.	④	⑤	④	⑤	②	①	③	②	③	④
No.	21	22	23	24	25	26	27	28	29	30
Ans.	①	⑤	④	⑤	⑤	③	②	③	⑤	④

Math & Physics Examination(TYPE A)

< Multiple choice Types > There is only one correct answer for each question. Mark your choice on the OMR answer sheet.

- The points for each question are listed next to the question number.
- You can use the right side of each page for your memo.

1. [2 points]

Compute $\frac{1}{\sqrt{8}-\sqrt{7}} - \frac{1}{\sqrt{8}+\sqrt{7}}$.

- ① $2\sqrt{7}$ ② $2\sqrt{8}$ ③ $\sqrt{7} + \sqrt{8}$
 ④ $\sqrt{8} - \sqrt{7}$ ⑤ $\sqrt{7} + \sqrt{8} + 1$

2. [2 points]

Compute $\sqrt{5+\sqrt{24}} + \sqrt{5-\sqrt{24}}$.

- ① $2\sqrt{2}$ ② $2\sqrt{3}$ ③ $\sqrt{2} + \sqrt{3}$
 ④ $\sqrt{2} + \sqrt{6}$ ⑤ $\sqrt{3} + \sqrt{6}$

3. [2 points]

Find $\sum_{k=1}^{50} \frac{1}{k(k+1)}$.

- ① $\frac{35}{51}$ ② $\frac{40}{51}$ ③ $\frac{45}{51}$ ④ $\frac{50}{51}$ ⑤ $\frac{55}{51}$

4. [2 points]

When $7^x = \sqrt{3}$, find $\frac{7^x + 7^{-x}}{7^x - 7^{-3x}}$.

- ① 1 ② $\frac{1}{2}$ ③ $\frac{3}{2}$ ④ $\frac{3}{4}$ ⑤ $\frac{5}{4}$

5. [2 points]

When $\log_{27}a = \frac{2}{3}$, find a .

- ① 1 ② 2 ③ 3 ④ 6 ⑤ 9

6. [2 points]

When α, β, γ are the solutions of

$x^3 - x^2 + 5x - 4 = 0$, find $\frac{1}{\alpha\beta} + \frac{1}{\beta\gamma} + \frac{1}{\gamma\alpha}$.

- ① 1 ② $\frac{1}{2}$ ③ $\frac{3}{2}$ ④ $\frac{1}{4}$ ⑤ $\frac{3}{4}$

7. [3 points]

When $z^3 = 1$ and $z \neq 1$, find $\frac{z^{18} + z^{17}}{z^{124}}$.

- ① i ② 1 ③ -1
 ④ $\frac{-1 + \sqrt{3}i}{2}$ ⑤ $\frac{-1 - \sqrt{3}i}{2}$

8. [3 points]

When $A = \begin{pmatrix} 1 & 3 \\ 0 & 1 \end{pmatrix}$ and $A^2 + A^3 + A^4 = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$, find $a + b + c + d$.

- ① 18 ② 23 ③ 28 ④ 33 ⑤ 38

9. [3 points]

Find the sum of all real solutions of $9^{x+1} - 3^{x+2} + 2 = 0$.

- ① $\log_3 2 - 2$ ② $\log_3 2 - 1$ ③ $\log_3 2$
 ④ $\log_3 2 + 1$ ⑤ $\log_3 2 + 2$

10. [3 points]

Find the sum of all solutions of $\cos 2x + 3\cos x + 2 = 0$ for $0 \leq x \leq 2\pi$.

- ① 2π ② 3π ③ 4π ④ 5π ⑤ 6π

11. [3 points]

When $\sin\theta$ and $\cos\theta$ are solutions of $3x^2 - 2x + b = 0$ for some θ , find b .

- ① $-\frac{1}{6}$ ② $-\frac{1}{3}$ ③ $-\frac{1}{2}$ ④ $-\frac{2}{3}$ ⑤ $-\frac{5}{6}$

12. [3 points]

When $\sin\alpha = \frac{4}{5}$ and $\sin\beta = \frac{12}{13}$ for

$0 < \alpha < \frac{\pi}{2}$, $0 < \beta < \frac{\pi}{2}$, find $\sin(\alpha + \beta)$.

- ① $\frac{44}{65}$ ② $\frac{48}{65}$ ③ $\frac{52}{65}$ ④ $\frac{56}{65}$ ⑤ $\frac{12}{13}$

13. [4 points]

Find $\lim_{x \rightarrow \infty} (\sqrt{x^2 + 5x - 3} - \sqrt{x^2 + 2x + 4})$.

- ① 1 ② $\frac{3}{2}$ ③ $\frac{5}{2}$ ④ $\frac{5}{4}$ ⑤ $\frac{7}{4}$

14. [4 points]

Find $\lim_{x \rightarrow 0} \frac{x}{\sin x + \sin 2x + \dots + \sin 9x}$.

- ① $\frac{1}{25}$ ② $\frac{1}{30}$ ③ $\frac{1}{35}$ ④ $\frac{1}{40}$ ⑤ $\frac{1}{45}$

15. [4 points]

When $f(x) = (2x^2 - 2x + 1)^2$, find $f'(1)$.

- ① 2 ② 4 ③ 6 ④ 8 ⑤ 10

16. [4 points]

When $y = ax + b$, for some constants a, b , is the tangent line to $y = \ln x$ at $x = 2$, find $a + b$.

- ① $\ln 2 - \frac{1}{2}$ ② $\ln 2 - \frac{1}{4}$ ③ $\ln 2$
④ $\ln 2 + \frac{1}{4}$ ⑤ $\ln 2 + \frac{1}{2}$

17. [4 points]

Find the shortest distance from $(-2, \frac{1}{2})$ to a point on the curve $y = x^2$.

- ① $\frac{\sqrt{5}}{8}$ ② $\frac{\sqrt{5}}{4}$ ③ $\frac{\sqrt{5}}{2}$ ④ $\sqrt{5}$ ⑤ $2\sqrt{5}$

18. [5 points]

When $\omega = \frac{\sqrt{4+2\sqrt{2}}}{i-1-\sqrt{2}}$, find ω^4 .

- ① 1 ② -1 ③ i ④ $-i$ ⑤ ω

19. [5 points]

Find $\int_0^{\ln 2} \frac{1}{1+e^x} dx$.

- ① $2\ln 2 - \ln 3$ ② $4\ln 2 - \ln 3$ ③ $6\ln 2 - \ln 3$
④ $8\ln 2 - \ln 3$ ⑤ $10\ln 2 - \ln 3$

20. [5 points]

Find the area of the region bounded by $y = \ln x$ ($x \geq 1$), $y = 0$ and the tangent line to $y = \ln x$ at $x = e$.

- ① $\frac{e-2}{2}$ ② $\frac{e-1}{2}$ ③ $\frac{e}{2}$ ④ $\frac{e+1}{2}$ ⑤ $\frac{e+2}{2}$

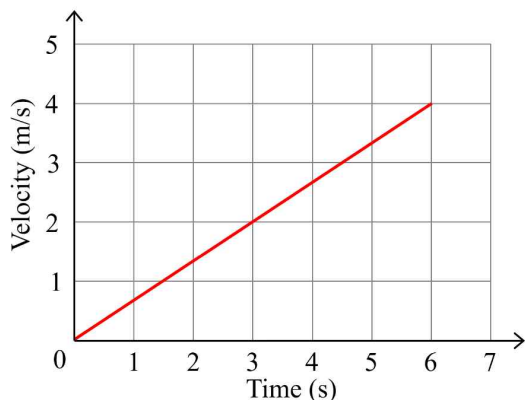
21. [5 points]

When $f(x)$ has a continuous derivative and $f(x) = 2 \int_e^x f'(t) dt - x \ln x$ for $x > 0$, find $f(e^2)$.

- ① $\frac{2e^2}{3} - 2e$ ② $e^2 - 2e$ ③ $\frac{4e^2}{3} - 2e$
④ $\frac{5e^2}{3} - 2e$ ⑤ $2e^2 - 2e$

22. [3 points]

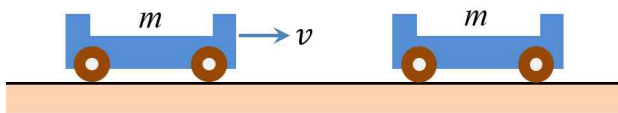
When a constant force was applied to an object with a mass of 3 kg, the speed of the object in a straight line changed as shown in the figure below. What is the magnitude of the force applied to this object?



- ① 1 N ② 1.5 N ③ 2 N
 ④ 2.5 N ⑤ 3 N

23. [3 points]

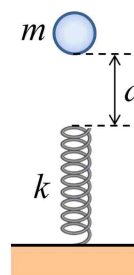
As shown in the figure below, a cart of mass m is moving at a speed v on a frictionless horizontal surface. Then, it collides with another cart of mass m that is stationary. The carts then become one body and move together along the same straight line. What is the speed of the carts that has become one body after the collision?



- ① v ② $\frac{v}{2}$ ③ $\frac{v}{3}$
 ④ $\frac{v}{4}$ ⑤ $\frac{2v}{3}$

24. [5 points]

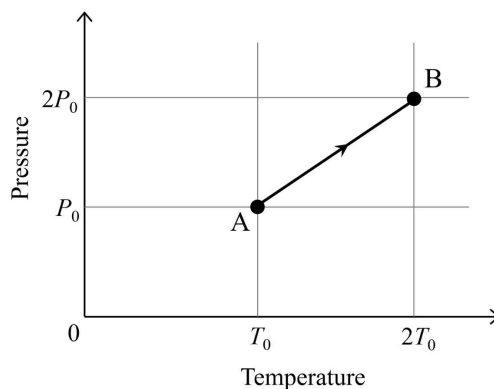
As shown in the figure, a mass $m = 1.0$ kg object is dropped from a vertical height $d = 1.0$ m above the spring to compress the spring with a spring constant $k = 175$ N/m. What will be the maximum compression of the spring? (Assume that the magnitude of the gravitational acceleration is $g = 10$ m/s² and all friction and air resistance are ignored.)



- ① 0.4 m ② 0.5 m ③ 0.6 m
 ④ 0.8 m ⑤ 0.9 m

25. [3 points]

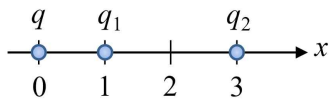
The figure below shows the relationship between pressure and temperature as the state of n moles of ideal gas changes from A to B. How much work does the gas do on the outside during the process A→B? (Where R is the gas constant.)



- ① $\frac{1}{2}P_0T_0$ ② $\frac{3}{2}P_0T_0$ ③ nRT_0
 ④ $\frac{3}{2}nRT_0$ ⑤ 0

26. [4 points]

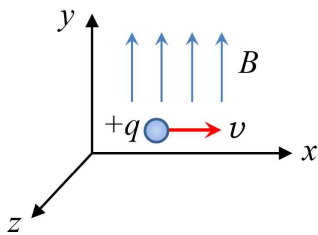
In the figure below, charges q_1 and q_2 are fixed at positions $x = 1$ and $x = 3$ respectively, and when a charge q is placed at position $x = 0$, no force is applied to the charge q . What is the ratio q_2/q_1 of the two charges?



- ① -3 ② 3 ③ 6
- ④ -9 ⑤ 9

27. [3 points]

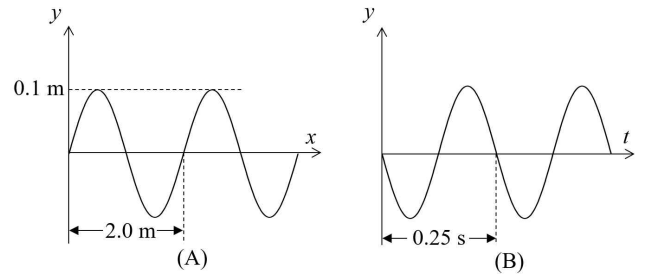
As shown in the figure below, there is a positive charge $+q$ moving in the $+x$ direction in a space where there is a magnetic field (B) in the $+y$ direction. What is the direction of the force applied to this charge? (Here, gravity is ignored.)



- ① $+x$ ② $-x$ ③ $+y$
- ④ $-y$ ⑤ $+z$

28. [3 points]

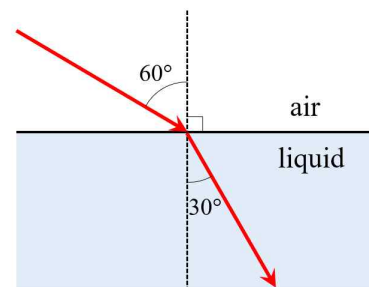
Figure (A) below shows the displacement (y) of a wave on a string as a function of position (x) at an instant, and Figure (B) shows the displacement (y) at a fixed point on the string as a function of time (t). What is the traveling speed of this wave?



- ① 2.0 m/s ② 4.0 m/s ③ 8.0 m/s
- ④ 10 m/s ⑤ 12 m/s

29. [3 points]

The figure below shows the path of light as it travels from air to a liquid. Assuming that the refractive index of air is $n_{\text{air}} = 1$, what is the refractive index of this liquid?



- ① $\sqrt{2}$ ② $\sqrt{3}$ ③ 2
- ④ 3 ⑤ 1.5

30. [3 points]

What is the energy of a photon of light with a wavelength of 300 nm? (Calculate using Planck's constant $h = 6.6 \times 10^{-34} \text{ J} \cdot \text{s}$ and the speed of light $c = 3.0 \times 10^8 \text{ m/s}$.)

- ① $6.6 \times 10^{-19} \text{ J}$ ② $4.4 \times 10^{-19} \text{ J}$
③ $3.3 \times 10^{-19} \text{ J}$ ④ $2.2 \times 10^{-19} \text{ J}$
⑤ $1.1 \times 10^{-19} \text{ J}$

Math & Physics Examination(TYPE B)

< Multiple choice Types > There is only one correct answer for each question. Mark your choice on the OMR answer sheet.

- The points for each question are listed next to the question number.
- You can use the right side of each page for your memo.

1. [2 points]

When α, β, γ are the solutions of

$$x^3 - x^2 + 5x - 4 = 0, \text{ find } \frac{1}{\alpha\beta} + \frac{1}{\beta\gamma} + \frac{1}{\gamma\alpha}.$$

- ① 1 ② $\frac{1}{2}$ ③ $\frac{3}{2}$ ④ $\frac{1}{4}$ ⑤ $\frac{3}{4}$

2. [2 points]

Compute $\frac{1}{\sqrt{8} - \sqrt{7}} - \frac{1}{\sqrt{8} + \sqrt{7}}.$

- ① $2\sqrt{7}$ ② $2\sqrt{8}$ ③ $\sqrt{7} + \sqrt{8}$
 ④ $\sqrt{8} - \sqrt{7}$ ⑤ $\sqrt{7} + \sqrt{8} + 1$

3. [2 points]

Compute $\sqrt{5 + \sqrt{24}} + \sqrt{5 - \sqrt{24}}.$

- ① $2\sqrt{2}$ ② $2\sqrt{3}$ ③ $\sqrt{2} + \sqrt{3}$
 ④ $\sqrt{2} + \sqrt{6}$ ⑤ $\sqrt{3} + \sqrt{6}$

4. [2 points]

Find $\sum_{k=1}^{50} \frac{1}{k(k+1)}.$

- ① $\frac{35}{51}$ ② $\frac{40}{51}$ ③ $\frac{45}{51}$ ④ $\frac{50}{51}$ ⑤ $\frac{55}{51}$

5. [2 points]

When $7^x = \sqrt{3}$, find $\frac{7^x + 7^{-x}}{7^x - 7^{-3x}}.$

- ① 1 ② $\frac{1}{2}$ ③ $\frac{3}{2}$ ④ $\frac{3}{4}$ ⑤ $\frac{5}{4}$

6. [2 points]

When $\log_{27} a = \frac{2}{3}$, find $a.$

- ① 1 ② 2 ③ 3 ④ 6 ⑤ 9

7. [3 points]

When $\sin \alpha = \frac{4}{5}$ and $\sin \beta = \frac{12}{13}$ for

$0 < \alpha < \frac{\pi}{2}, 0 < \beta < \frac{\pi}{2}$, find $\sin(\alpha + \beta).$

- ① $\frac{44}{65}$ ② $\frac{48}{65}$ ③ $\frac{52}{65}$ ④ $\frac{56}{65}$ ⑤ $\frac{12}{13}$

8. [3 points]

When $z^3 = 1$ and $z \neq 1$, find $\frac{z^{18} + z^{17}}{z^{124}}.$

- ① i ② 1 ③ -1
 ④ $\frac{-1 + \sqrt{3}i}{2}$ ⑤ $\frac{-1 - \sqrt{3}i}{2}$

9. [3 points]

When $A = \begin{pmatrix} 1 & 3 \\ 0 & 1 \end{pmatrix}$ and $A^2 + A^3 + A^4 = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$,

find $a + b + c + d.$

- ① 18 ② 23 ③ 28 ④ 33 ⑤ 38

10. [3 points]

Find the sum of all real solutions of

$$9^{x+1} - 3^{x+2} + 2 = 0.$$

- ① $\log_3 2 - 2$ ② $\log_3 2 - 1$ ③ $\log_3 2$
 ④ $\log_3 2 + 1$ ⑤ $\log_3 2 + 2$

11. [3 points]

Find the sum of all solutions of

$$\cos 2x + 3\cos x + 2 = 0 \text{ for } 0 \leq x \leq 2\pi.$$

- ① 2π ② 3π ③ 4π ④ 5π ⑤ 6π

12. [3 points]

When $\sin \theta$ and $\cos \theta$ are solutions of

$$3x^2 - 2x + b = 0 \text{ for some } \theta, \text{ find } b.$$

- ① $-\frac{1}{6}$ ② $-\frac{1}{3}$ ③ $-\frac{1}{2}$ ④ $-\frac{2}{3}$ ⑤ $-\frac{5}{6}$

13. [4 points]

Find the shortest distance from $(-2, \frac{1}{2})$ to a point on the curve $y = x^2$.

- ① $\frac{\sqrt{5}}{8}$ ② $\frac{\sqrt{5}}{4}$ ③ $\frac{\sqrt{5}}{2}$ ④ $\sqrt{5}$ ⑤ $2\sqrt{5}$

14. [4 points]

$$\text{Find } \lim_{x \rightarrow \infty} (\sqrt{x^2 + 5x - 3} - \sqrt{x^2 + 2x + 4}).$$

- ① 1 ② $\frac{3}{2}$ ③ $\frac{5}{2}$ ④ $\frac{5}{4}$ ⑤ $\frac{7}{4}$

15. [4 points]

$$\text{Find } \lim_{x \rightarrow 0} \frac{x}{\sin x + \sin 2x + \dots + \sin 9x}.$$

- ① $\frac{1}{25}$ ② $\frac{1}{30}$ ③ $\frac{1}{35}$ ④ $\frac{1}{40}$ ⑤ $\frac{1}{45}$

16. [4 points]

When $f(x) = (2x^2 - 2x + 1)^2$, find $f'(1)$.

- ① 2 ② 4 ③ 6 ④ 8 ⑤ 10

17. [4 points]

When $y = ax + b$, for some constants a, b , is

the tangent line to $y = \ln x$ at $x = 2$, find $a + b$.

- ① $\ln 2 - \frac{1}{2}$ ② $\ln 2 - \frac{1}{4}$ ③ $\ln 2$
④ $\ln 2 + \frac{1}{4}$ ⑤ $\ln 2 + \frac{1}{2}$

18. [5 points]

When $f(x)$ has a continuous derivative and

$$f(x) = 2 \int_e^x f'(t) dt - x \ln x \text{ for } x > 0, \text{ find } f(e^2).$$

- ① $\frac{2e^2}{3} - 2e$ ② $e^2 - 2e$ ③ $\frac{4e^2}{3} - 2e$
④ $\frac{5e^2}{3} - 2e$ ⑤ $2e^2 - 2e$

19. [5 points]

$$\text{When } \omega = \frac{\sqrt{4+2\sqrt{2}}}{i-1-\sqrt{2}}, \text{ find } \omega^4.$$

- ① 1 ② -1 ③ i ④ $-i$ ⑤ ω

20. [5 points]

$$\text{Find } \int_0^{\ln 2} \frac{1}{1+e^x} dx.$$

- ① $2\ln 2 - \ln 3$ ② $4\ln 2 - \ln 3$ ③ $6\ln 2 - \ln 3$
④ $8\ln 2 - \ln 3$ ⑤ $10\ln 2 - \ln 3$

21. [5 points]

Find the area of the region bounded by

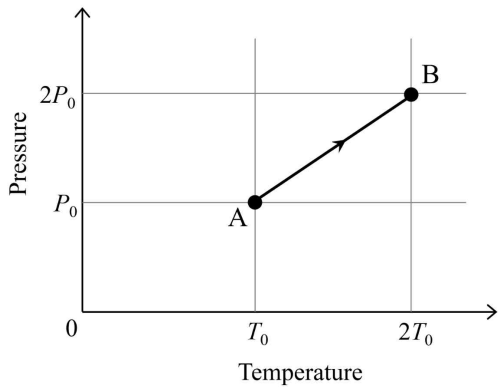
$y = \ln x (x \geq 1)$, $y = 0$ and the tangent line to

$y = \ln x$ at $x = e$.

- ① $\frac{e-2}{2}$ ② $\frac{e-1}{2}$ ③ $\frac{e}{2}$ ④ $\frac{e+1}{2}$ ⑤ $\frac{e+2}{2}$

22. [3 points]

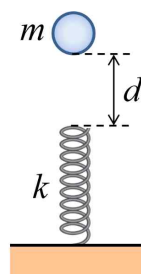
The figure below shows the relationship between pressure and temperature as the state of n moles of ideal gas changes from A to B. How much work does the gas do on the outside during the process A \rightarrow B? (Where R is the gas constant.)



- ① $\frac{1}{2}P_0T_0$ ② $\frac{3}{2}P_0T_0$ ③ nRT_0
 ④ $\frac{3}{2}nRT_0$ ⑤ 0

23. [5 points]

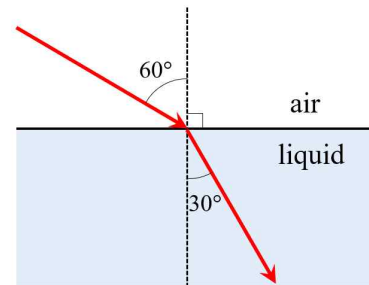
As shown in the figure, a mass $m = 1.0$ kg object is dropped from a vertical height $d = 1.0$ m above the spring to compress the spring with a spring constant $k = 175$ N/m. What will be the maximum compression of the spring? (Assume that the magnitude of the gravitational acceleration is $g = 10$ m/s² and all friction and air resistance are ignored.)



- ① 0.4 m ② 0.5 m ③ 0.6 m
 ④ 0.8 m ⑤ 0.9 m

24. [3 points]

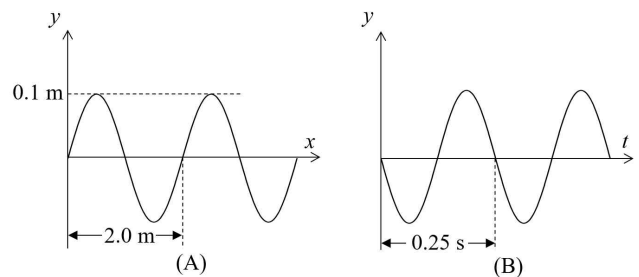
The figure below shows the path of light as it travels from air to a liquid. Assuming that the refractive index of air is $n_{\text{air}} = 1$, what is the refractive index of this liquid?



- ① $\sqrt{2}$ ② $\sqrt{3}$ ③ 2
 ④ 3 ⑤ 1.5

25. [3 points]

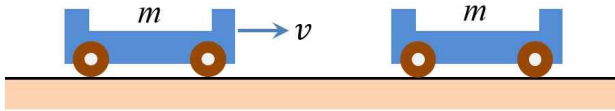
Figure (A) below shows the displacement (y) of a wave on a string as a function of position (x) at an instant, and Figure (B) shows the displacement (y) at a fixed point on the string as a function of time (t). What is the traveling speed of this wave?



- ① 2.0 m/s ② 4.0 m/s ③ 8.0 m/s
 ④ 10 m/s ⑤ 12 m/s

26. [3 points]

As shown in the figure below, a cart of mass m is moving at a speed v on a frictionless horizontal surface. Then, it collides with another cart of mass m that is stationary. The carts then become one body and move together along the same straight line. What is the speed of the carts that has become one body after the collision?



- ① v ② $\frac{v}{2}$ ③ $\frac{v}{3}$
- ④ $\frac{v}{4}$ ⑤ $\frac{2v}{3}$

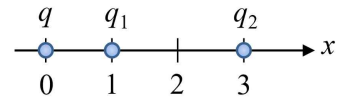
27. [3 points]

What is the energy of a photon of light with a wavelength of 300 nm? (Calculate using Planck's constant $h = 6.6 \times 10^{-34} \text{ J} \cdot \text{s}$ and the speed of light $c = 3.0 \times 10^8 \text{ m/s}$.)

- ① $6.6 \times 10^{-19} \text{ J}$ ② $4.4 \times 10^{-19} \text{ J}$
- ③ $3.3 \times 10^{-19} \text{ J}$ ④ $2.2 \times 10^{-19} \text{ J}$
- ⑤ $1.1 \times 10^{-19} \text{ J}$

28. [4 points]

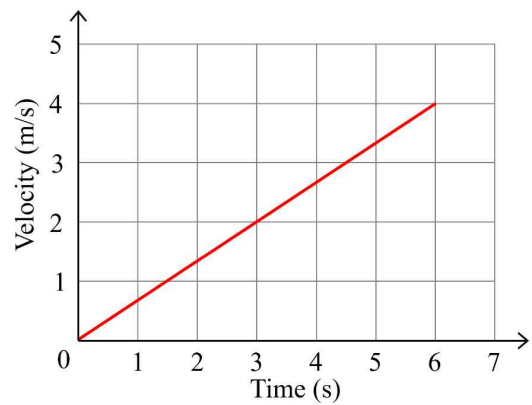
In the figure below, charges q_1 and q_2 are fixed at positions $x = 1$ and $x = 3$ respectively, and when a charge q is placed at position $x = 0$, no force is applied to the charge q . What is the ratio q_2/q_1 of the two charges?



- ① -3 ② 3 ③ 6
- ④ -9 ⑤ 9

29. [3 points]

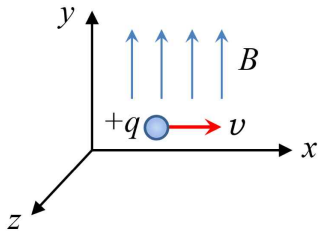
When a constant force was applied to an object with a mass of 3 kg, the speed of the object in a straight line changed as shown in the figure below. What is the magnitude of the force applied to this object?



- ① 1 N ② 1.5 N ③ 2 N
- ④ 2.5 N ⑤ 3 N

30. [3 points]

As shown in the figure below, there is a positive charge $+q$ moving in the $+x$ direction in a space where there is a magnetic field (B) in the $+y$ direction. What is the direction of the force applied to this charge? (Here, gravity is ignored.)



- ① $+x$ ② $-x$ ③ $+y$
④ $-y$ ⑤ $+z$

Math & Physics Examination(TYPE C)

< Multiple choice Types > There is only one correct answer for each question. Mark your choice on the OMR answer sheet.

- The points for each question are listed next to the question number.
- You can use the right side of each page for your memo.

1. [2 points]

When $\log_{27}a = \frac{2}{3}$, find a .

- ① 1 ② 2 ③ 3 ④ 6 ⑤ 9

2. [2 points]

When α, β, γ are the solutions of

$x^3 - x^2 + 5x - 4 = 0$, find $\frac{1}{\alpha\beta} + \frac{1}{\beta\gamma} + \frac{1}{\gamma\alpha}$.

- ① 1 ② $\frac{1}{2}$ ③ $\frac{3}{2}$ ④ $\frac{1}{4}$ ⑤ $\frac{3}{4}$

3. [2 points]

Compute $\frac{1}{\sqrt{8}-\sqrt{7}} - \frac{1}{\sqrt{8}+\sqrt{7}}$.

- ① $2\sqrt{7}$ ② $2\sqrt{8}$ ③ $\sqrt{7} + \sqrt{8}$
 ④ $\sqrt{8} - \sqrt{7}$ ⑤ $\sqrt{7} + \sqrt{8} + 1$

4. [2 points]

Compute $\sqrt{5+\sqrt{24}} + \sqrt{5-\sqrt{24}}$.

- ① $2\sqrt{2}$ ② $2\sqrt{3}$ ③ $\sqrt{2} + \sqrt{3}$
 ④ $\sqrt{2} + \sqrt{6}$ ⑤ $\sqrt{3} + \sqrt{6}$

5. [2 points]

Find $\sum_{k=1}^{50} \frac{1}{k(k+1)}$.

- ① $\frac{35}{51}$ ② $\frac{40}{51}$ ③ $\frac{45}{51}$ ④ $\frac{50}{51}$ ⑤ $\frac{55}{51}$

6. [2 points]

When $7^x = \sqrt{3}$, find $\frac{7^x + 7^{-x}}{7^x - 7^{-3x}}$.

- ① 1 ② $\frac{1}{2}$ ③ $\frac{3}{2}$ ④ $\frac{3}{4}$ ⑤ $\frac{5}{4}$

7. [3 points]

When $\sin\theta$ and $\cos\theta$ are solutions of $3x^2 - 2x + b = 0$ for some θ , find b .

- ① $-\frac{1}{6}$ ② $-\frac{1}{3}$ ③ $-\frac{1}{2}$ ④ $-\frac{2}{3}$ ⑤ $-\frac{5}{6}$

8. [3 points]

When $\sin\alpha = \frac{4}{5}$ and $\sin\beta = \frac{12}{13}$ for

$0 < \alpha < \frac{\pi}{2}$, $0 < \beta < \frac{\pi}{2}$, find $\sin(\alpha + \beta)$.

- ① $\frac{44}{65}$ ② $\frac{48}{65}$ ③ $\frac{52}{65}$ ④ $\frac{56}{65}$ ⑤ $\frac{12}{13}$

9. [3 points]

When $z^3 = 1$ and $z \neq 1$, find $\frac{z^{18} + z^{17}}{z^{124}}$.

- ① i ② 1 ③ -1
 ④ $\frac{-1 + \sqrt{3}i}{2}$ ⑤ $\frac{-1 - \sqrt{3}i}{2}$

10. [3 points]

When $A = \begin{pmatrix} 1 & 3 \\ 0 & 1 \end{pmatrix}$ and $A^2 + A^3 + A^4 = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$,

find $a + b + c + d$.

- ① 18 ② 23 ③ 28 ④ 33 ⑤ 38

11. [3 points]

Find the sum of all real solutions of

$$9^{x+1} - 3^{x+2} + 2 = 0.$$

- ① $\log_3 2 - 2$ ② $\log_3 2 - 1$ ③ $\log_3 2$
④ $\log_3 2 + 1$ ⑤ $\log_3 2 + 2$

12. [3 points]

Find the sum of all solutions of

$$\cos 2x + 3\cos x + 2 = 0 \text{ for } 0 \leq x \leq 2\pi.$$

- ① 2π ② 3π ③ 4π ④ 5π ⑤ 6π

13. [4 points]

When $y = ax + b$, for some constants a, b , is the tangent line to $y = \ln x$ at $x = 2$, find $a + b$.

- ① $\ln 2 - \frac{1}{2}$ ② $\ln 2 - \frac{1}{4}$ ③ $\ln 2$
④ $\ln 2 + \frac{1}{4}$ ⑤ $\ln 2 + \frac{1}{2}$

14. [4 points]

Find the shortest distance from $(-2, \frac{1}{2})$ to a point on the curve $y = x^2$.

- ① $\frac{\sqrt{5}}{8}$ ② $\frac{\sqrt{5}}{4}$ ③ $\frac{\sqrt{5}}{2}$ ④ $\sqrt{5}$ ⑤ $2\sqrt{5}$

15. [4 points]

Find $\lim_{x \rightarrow \infty} (\sqrt{x^2 + 5x - 3} - \sqrt{x^2 + 2x + 4})$.

- ① 1 ② $\frac{3}{2}$ ③ $\frac{5}{2}$ ④ $\frac{5}{4}$ ⑤ $\frac{7}{4}$

16. [4 points]

Find $\lim_{x \rightarrow 0} \frac{x}{\sin x + \sin 2x + \dots + \sin 9x}$.

- ① $\frac{1}{25}$ ② $\frac{1}{30}$ ③ $\frac{1}{35}$ ④ $\frac{1}{40}$ ⑤ $\frac{1}{45}$

17. [4 points]

When $f(x) = (2x^2 - 2x + 1)^2$, find $f'(1)$.

- ① 2 ② 4 ③ 6 ④ 8 ⑤ 10

18. [5 points]

Find the area of the region bounded by

$y = \ln x$ ($x \geq 1$), $y = 0$ and the tangent line to $y = \ln x$ at $x = e$.

- ① $\frac{e-2}{2}$ ② $\frac{e-1}{2}$ ③ $\frac{e}{2}$ ④ $\frac{e+1}{2}$ ⑤ $\frac{e+2}{2}$

19. [5 points]

When $f(x)$ has a continuous derivative and

$f(x) = 2 \int_e^x f'(t) dt - x \ln x$ for $x > 0$, find $f(e^2)$.

- ① $\frac{2e^2}{3} - 2e$ ② $e^2 - 2e$ ③ $\frac{4e^2}{3} - 2e$
④ $\frac{5e^2}{3} - 2e$ ⑤ $2e^2 - 2e$

20. [5 points]

When $\omega = \frac{\sqrt{4+2\sqrt{2}}}{i-1-\sqrt{2}}$, find ω^4 .

- ① 1 ② -1 ③ i ④ $-i$ ⑤ ω

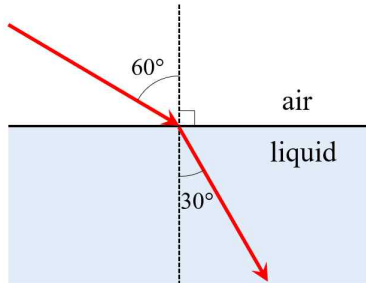
21. [5 points]

Find $\int_0^{\ln 2} \frac{1}{1+e^x} dx$.

- ① $2\ln 2 - \ln 3$ ② $4\ln 2 - \ln 3$ ③ $6\ln 2 - \ln 3$
④ $8\ln 2 - \ln 3$ ⑤ $10\ln 2 - \ln 3$

22. [3 points]

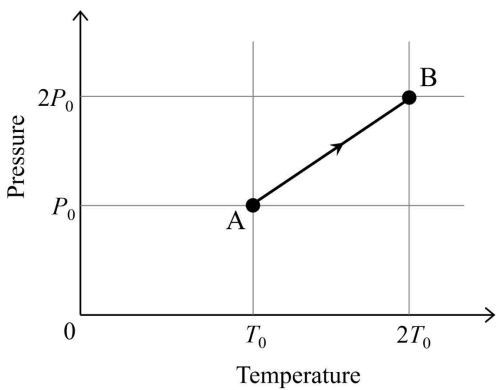
The figure below shows the path of light as it travels from air to a liquid. Assuming that the refractive index of air is $n_{\text{air}} = 1$, what is the refractive index of this liquid?



- ① $\sqrt{2}$ ② $\sqrt{3}$ ③ 2
 ④ 3 ⑤ 1.5

23. [3 points]

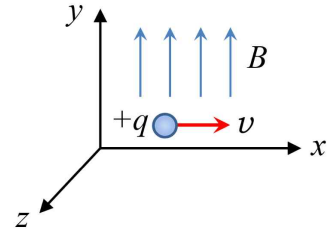
The figure below shows the relationship between pressure and temperature as the state of n moles of ideal gas changes from A to B. How much work does the gas do on the outside during the process A \rightarrow B? (Where R is the gas constant.)



- ① $\frac{1}{2}P_0T_0$ ② $\frac{3}{2}P_0T_0$ ③ nRT_0
 ④ $\frac{3}{2}nRT_0$ ⑤ 0

24. [3 points]

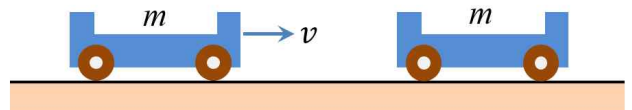
As shown in the figure below, there is a positive charge $+q$ moving in the $+x$ direction in a space where there is a magnetic field (B) in the $+y$ direction. What is the direction of the force applied to this charge? (Here, gravity is ignored.)



- ① $+x$ ② $-x$ ③ $+y$
 ④ $-y$ ⑤ $+z$

25. [3 points]

As shown in the figure below, a cart of mass m is moving at a speed v on a frictionless horizontal surface. Then, it collides with another cart of mass m that is stationary. The carts then become one body and move together along the same straight line. What is the speed of the carts that has become one body after the collision?



- ① v ② $\frac{v}{2}$ ③ $\frac{v}{3}$
 ④ $\frac{v}{4}$ ⑤ $\frac{2v}{3}$

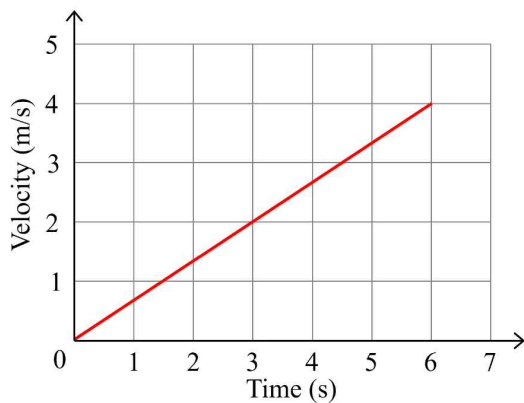
26. [3 points]

What is the energy of a photon of light with a wavelength of 300 nm ? (Calculate using Planck's constant $h = 6.6 \times 10^{-34} \text{ J} \cdot \text{s}$ and the speed of light $c = 3.0 \times 10^8 \text{ m/s}$.)

- ① $6.6 \times 10^{-19} \text{ J}$ ② $4.4 \times 10^{-19} \text{ J}$
- ③ $3.3 \times 10^{-19} \text{ J}$ ④ $2.2 \times 10^{-19} \text{ J}$
- ⑤ $1.1 \times 10^{-19} \text{ J}$

27. [3 points]

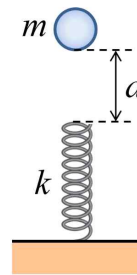
When a constant force was applied to an object with a mass of 3 kg , the speed of the object in a straight line changed as shown in the figure below. What is the magnitude of the force applied to this object?



- ① 1 N ② 1.5 N ③ 2 N
- ④ 2.5 N ⑤ 3 N

28. [5 points]

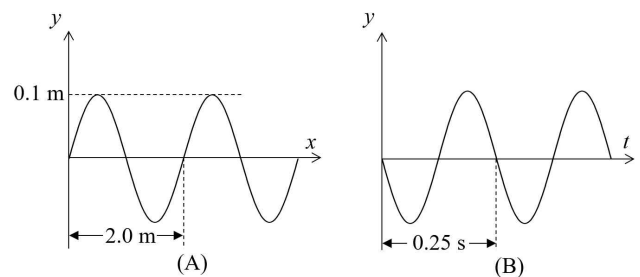
As shown in the figure, a mass $m = 1.0 \text{ kg}$ object is dropped from a vertical height $d = 1.0 \text{ m}$ above the spring to compress the spring with a spring constant $k = 175 \text{ N/m}$. What will be the maximum compression of the spring? (Assume that the magnitude of the gravitational acceleration is $g = 10 \text{ m/s}^2$ and all friction and air resistance are ignored.)



- ① 0.4 m ② 0.5 m ③ 0.6 m
- ④ 0.8 m ⑤ 0.9 m

29. [3 points]

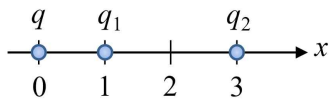
Figure (A) below shows the displacement (y) of a wave on a string as a function of position (x) at an instant, and Figure (B) shows the displacement (y) at a fixed point on the string as a function of time (t). What is the traveling speed of this wave?



- ① 2.0 m/s ② 4.0 m/s ③ 8.0 m/s
- ④ 10 m/s ⑤ 12 m/s

30. [4 points]

In the figure below, charges q_1 and q_2 are fixed at positions $x = 1$ and $x = 3$ respectively, and when a charge q is placed at position $x = 0$, no force is applied to the charge q . What is the ratio q_2/q_1 of the two charges?



① -3

② 3

③ 6

④ -9

⑤ 9

Math & Physics Examination(TYPE D)

< Multiple choice Types > There is only one correct answer for each question. Mark your choice on the OMR answer sheet.

- The points for each question are listed next to the question number.
- You can use the right side of each page for your memo.

1. [2 points]

Find $\sum_{k=1}^{50} \frac{1}{k(k+1)}$.

- ① $\frac{35}{51}$ ② $\frac{40}{51}$ ③ $\frac{45}{51}$ ④ $\frac{50}{51}$ ⑤ $\frac{55}{51}$

2. [2 points]

When $\log_{27} a = \frac{2}{3}$, find a .

- ① 1 ② 2 ③ 3 ④ 6 ⑤ 9

3. [2 points]

When α, β, γ are the solutions of

$x^3 - x^2 + 5x - 4 = 0$, find $\frac{1}{\alpha\beta} + \frac{1}{\beta\gamma} + \frac{1}{\gamma\alpha}$.

- ① 1 ② $\frac{1}{2}$ ③ $\frac{3}{2}$ ④ $\frac{1}{4}$ ⑤ $\frac{3}{4}$

4. [2 points]

Compute $\frac{1}{\sqrt{8}-\sqrt{7}} - \frac{1}{\sqrt{8}+\sqrt{7}}$.

- ① $2\sqrt{7}$ ② $2\sqrt{8}$ ③ $\sqrt{7} + \sqrt{8}$
 ④ $\sqrt{8} - \sqrt{7}$ ⑤ $\sqrt{7} + \sqrt{8} + 1$

5. [2 points]

Compute $\sqrt{5+\sqrt{24}} + \sqrt{5-\sqrt{24}}$.

- ① $2\sqrt{2}$ ② $2\sqrt{3}$ ③ $\sqrt{2} + \sqrt{3}$
 ④ $\sqrt{2} + \sqrt{6}$ ⑤ $\sqrt{3} + \sqrt{6}$

6. [2 points]

When $7^x = \sqrt{3}$, find $\frac{7^x + 7^{-x}}{7^x - 7^{-3x}}$.

- ① 1 ② $\frac{1}{2}$ ③ $\frac{3}{2}$ ④ $\frac{3}{4}$ ⑤ $\frac{5}{4}$

7. [3 points]

Find the sum of all solutions of

$\cos 2x + 3\cos x + 2 = 0$ for $0 \leq x \leq 2\pi$.

- ① 2π ② 3π ③ 4π ④ 5π ⑤ 6π

8. [3 points]

When $\sin \theta$ and $\cos \theta$ are solutions of

$3x^2 - 2x + b = 0$ for some θ , find b .

- ① $-\frac{1}{6}$ ② $-\frac{1}{3}$ ③ $-\frac{1}{2}$ ④ $-\frac{2}{3}$ ⑤ $-\frac{5}{6}$

9. [3 points]

When $\sin \alpha = \frac{4}{5}$ and $\sin \beta = \frac{12}{13}$ for

$0 < \alpha < \frac{\pi}{2}$, $0 < \beta < \frac{\pi}{2}$, find $\sin(\alpha + \beta)$.

- ① $\frac{44}{65}$ ② $\frac{48}{65}$ ③ $\frac{52}{65}$ ④ $\frac{56}{65}$ ⑤ $\frac{12}{13}$

10. [3 points]

When $z^3 = 1$ and $z \neq 1$, find $\frac{z^{18} + z^{17}}{z^{124}}$.

- ① i ② 1 ③ -1
 ④ $\frac{-1 + \sqrt{3}i}{2}$ ⑤ $\frac{-1 - \sqrt{3}i}{2}$

11. [3 points]

Find the sum of all real solutions of

$$9^{x+1} - 3^{x+2} + 2 = 0.$$

- ① $\log_3 2 - 2$ ② $\log_3 2 - 1$ ③ $\log_3 2$
④ $\log_3 2 + 1$ ⑤ $\log_3 2 + 2$

12. [3 points]

When $A = \begin{pmatrix} 1 & 3 \\ 0 & 1 \end{pmatrix}$ and $A^2 + A^3 + A^4 = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$,

find $a + b + c + d$.

- ① 18 ② 23 ③ 28 ④ 33 ⑤ 38

13. [4 points]

When $f(x) = (2x^2 - 2x + 1)^2$, find $f'(1)$.

- ① 2 ② 4 ③ 6 ④ 8 ⑤ 10

14. [4 points]

When $y = ax + b$, for some constants a, b , is the tangent line to $y = \ln x$ at $x = 2$, find $a + b$.

- ① $\ln 2 - \frac{1}{2}$ ② $\ln 2 - \frac{1}{4}$ ③ $\ln 2$
④ $\ln 2 + \frac{1}{4}$ ⑤ $\ln 2 + \frac{1}{2}$

15. [4 points]

Find the shortest distance from $(-2, \frac{1}{2})$ to a point on the curve $y = x^2$.

- ① $\frac{\sqrt{5}}{8}$ ② $\frac{\sqrt{5}}{4}$ ③ $\frac{\sqrt{5}}{2}$ ④ $\sqrt{5}$ ⑤ $2\sqrt{5}$

16. [4 points]

Find $\lim_{x \rightarrow \infty} (\sqrt{x^2 + 5x - 3} - \sqrt{x^2 + 2x + 4})$.

- ① 1 ② $\frac{3}{2}$ ③ $\frac{5}{2}$ ④ $\frac{5}{4}$ ⑤ $\frac{7}{4}$

17. [4 points]

Find $\lim_{x \rightarrow 0} \frac{x}{\sin x + \sin 2x + \dots + \sin 9x}$.

- ① $\frac{1}{25}$ ② $\frac{1}{30}$ ③ $\frac{1}{35}$ ④ $\frac{1}{40}$ ⑤ $\frac{1}{45}$

18. [5 points]

Find $\int_0^{\ln 2} \frac{1}{1 + e^x} dx$.

- ① $2 \ln 2 - \ln 3$ ② $4 \ln 2 - \ln 3$ ③ $6 \ln 2 - \ln 3$
④ $8 \ln 2 - \ln 3$ ⑤ $10 \ln 2 - \ln 3$

19. [5 points]

Find the area of the region bounded by

$y = \ln x$ ($x \geq 1$), $y = 0$ and the tangent line to $y = \ln x$ at $x = e$.

- ① $\frac{e-2}{2}$ ② $\frac{e-1}{2}$ ③ $\frac{e}{2}$ ④ $\frac{e+1}{2}$ ⑤ $\frac{e+2}{2}$

20. [5 points]

When $f(x)$ has a continuous derivative and $f(x) = 2 \int_e^x f'(t) dt - x \ln x$ for $x > 0$, find $f(e^2)$.

- ① $\frac{2e^2}{3} - 2e$ ② $e^2 - 2e$ ③ $\frac{4e^2}{3} - 2e$
④ $\frac{5e^2}{3} - 2e$ ⑤ $2e^2 - 2e$

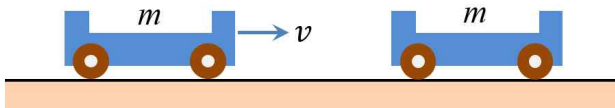
21. [5 points]

When $\omega = \frac{\sqrt{4+2\sqrt{2}}}{i-1-\sqrt{2}}$, find ω^4 .

- ① 1 ② -1 ③ i ④ $-i$ ⑤ ω

22. [3 points]

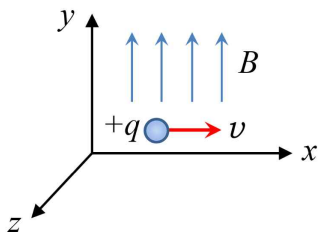
As shown in the figure below, a cart of mass m is moving at a speed v on a frictionless horizontal surface. Then, it collides with another cart of mass m that is stationary. The carts then become one body and move together along the same straight line. What is the speed of the carts that has become one body after the collision?



- ① v ② $\frac{v}{2}$ ③ $\frac{v}{3}$
- ④ $\frac{v}{4}$ ⑤ $\frac{2v}{3}$

23. [3 points]

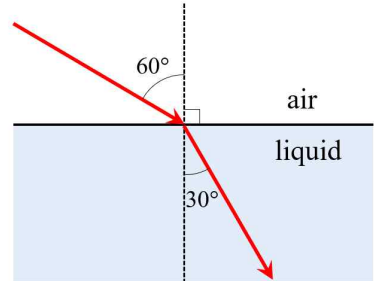
As shown in the figure below, there is a positive charge $+q$ moving in the $+x$ direction in a space where there is a magnetic field (B) in the $+y$ direction. What is the direction of the force applied to this charge? (Here, gravity is ignored.)



- ① $+x$ ② $-x$ ③ $+y$
- ④ $-y$ ⑤ $+z$

24. [3 points]

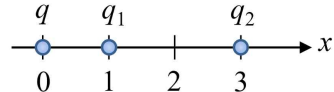
The figure below shows the path of light as it travels from air to a liquid. Assuming that the refractive index of air is $n_{\text{air}} = 1$, what is the refractive index of this liquid?



- ① $\sqrt{2}$ ② $\sqrt{3}$ ③ 2
- ④ 3 ⑤ 1.5

25. [4 points]

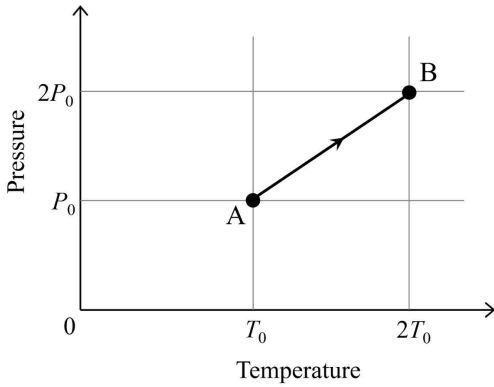
In the figure below, charges q_1 and q_2 are fixed at positions $x = 1$ and $x = 3$ respectively, and when a charge q is placed at position $x = 0$, no force is applied to the charge q . What is the ratio q_2/q_1 of the two charges?



- ① -3 ② 3 ③ 6
- ④ -9 ⑤ 9

26. [3 points]

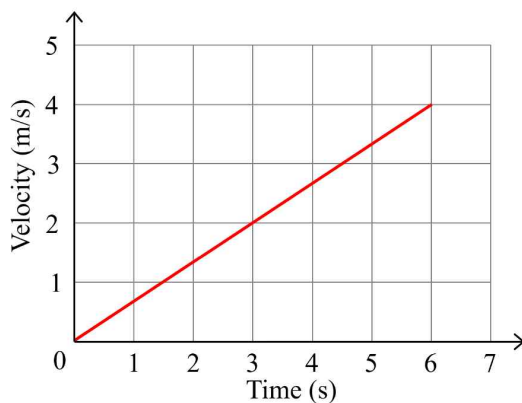
The figure below shows the relationship between pressure and temperature as the state of n moles of ideal gas changes from A to B. How much work does the gas do on the outside during the process A \rightarrow B? (Where R is the gas constant.)



- ① $\frac{1}{2}P_0T_0$ ② $\frac{3}{2}P_0T_0$ ③ nRT_0
 ④ $\frac{3}{2}nRT_0$ ⑤ 0

27. [3 points]

When a constant force was applied to an object with a mass of 3 kg, the speed of the object in a straight line changed as shown in the figure below. What is the magnitude of the force applied to this object?



- ① 1 N ② 1.5 N ③ 2 N
 ④ 2.5 N ⑤ 3 N

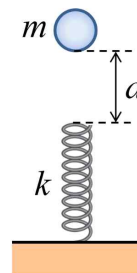
28. [3 points]

What is the energy of a photon of light with a wavelength of 300 nm? (Calculate using Planck's constant $h = 6.6 \times 10^{-34} \text{ J} \cdot \text{s}$ and the speed of light $c = 3.0 \times 10^8 \text{ m/s}$.)

- ① $6.6 \times 10^{-19} \text{ J}$ ② $4.4 \times 10^{-19} \text{ J}$
 ③ $3.3 \times 10^{-19} \text{ J}$ ④ $2.2 \times 10^{-19} \text{ J}$
 ⑤ $1.1 \times 10^{-19} \text{ J}$

29. [5 points]

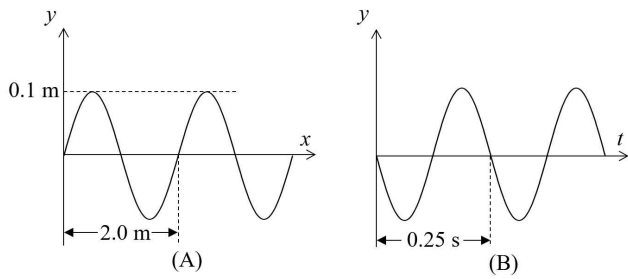
As shown in the figure, a mass $m = 1.0 \text{ kg}$ object is dropped from a vertical height $d = 1.0 \text{ m}$ above the spring to compress the spring with a spring constant $k = 175 \text{ N/m}$. What will be the maximum compression of the spring? (Assume that the magnitude of the gravitational acceleration is $g = 10 \text{ m/s}^2$ and all friction and air resistance are ignored.)



- ① 0.4 m ② 0.5 m ③ 0.6 m
 ④ 0.8 m ⑤ 0.9 m

30. [3 points]

Figure (A) below shows the displacement (y) of a wave on a string as a function of position (x) at an instant, and Figure (B) shows the displacement (y) at a fixed point on the string as a function of time (t). What is the traveling speed of this wave?



- ① 2.0 m/s ② 4.0 m/s ③ 8.0 m/s
④ 10 m/s ⑤ 12 m/s

2025 IUT 3rd SOCIE Answer Sheets

[TypeA]

No.	1	2	3	4	5	6	7	8	9	10
Ans.	①	②	④	③	⑤	④	③	④	①	②
No.	11	12	13	14	15	16	17	18	19	20
Ans.	⑤	④	②	⑤	②	①	③	③	①	①
No.	21	22	23	24	25	26	27	28	29	30
Ans.	⑤	③	②	①	⑤	④	⑤	③	②	①

[TypeB]

No.	1	2	3	4	5	6	7	8	9	10
Ans.	④	①	②	④	③	⑤	④	③	④	①
No.	11	12	13	14	15	16	17	18	19	20
Ans.	②	⑤	③	②	⑤	②	①	⑤	③	①
No.	21	22	23	24	25	26	27	28	29	30
Ans.	①	⑤	①	②	③	②	①	④	③	⑤

[TypeC]

No.	1	2	3	4	5	6	7	8	9	10
Ans.	⑤	④	①	②	④	③	⑤	④	③	④
No.	11	12	13	14	15	16	17	18	19	20
Ans.	①	②	①	③	②	⑤	②	①	⑤	③
No.	21	22	23	24	25	26	27	28	29	30
Ans.	①	②	⑤	⑤	②	①	③	①	③	④

[TypeD]

No.	1	2	3	4	5	6	7	8	9	10
Ans.	④	⑤	④	①	②	③	②	⑤	④	③
No.	11	12	13	14	15	16	17	18	19	20
Ans.	①	④	②	①	③	②	⑤	①	①	⑤
No.	21	22	23	24	25	26	27	28	29	30
Ans.	③	②	⑤	②	④	⑤	③	①	①	③